

Augmented Sculpture: Computer Ghosts of Physical Artifacts

Valery Adzhiev, Peter Comminos and Alexander Pasko

Valery Adzhiev (educator, researcher), The National Centre for Computer Animation, Bournemouth Media School, Bournemouth University, Talbot Campus, Poole, BH12 5BB, U.K.

E-mail: vadzhiev@bournemouth.ac.uk.

Peter Comminos (educator, researcher), The National Centre for Computer Animation, Bournemouth Media School, Bournemouth University, Talbot Campus, Poole, BH12 5BB, U.K.

E-mail: peterc@bournemouth.ac.uk.

Alexander Pasko (educator, researcher), Faculty of Computer and Information Sciences, Hosei University, 3-7-2 Kajino-cho, Koganei-shi, Tokyo, 184-8584, Japan.

E-mail: pasko@k.hosei.ac.jp.

ABSTRACT

This paper describes an approach to computer-based sculpting concerned with the creation and modification of digital models based on physical abstract sculptures. The authors begin by presenting a survey of current methods for the creation of computer-based sculpted artifacts. They proceed to present some original methods and tools based on the Function Representation of geometric models. They introduce a specialized computer language, called HyperFun, that facilitates the modeling of complex objects. In addition to presenting computer-generated textured and animated models of pre-existing sculptures, they also show how novel shapes can be generated using the HyperFun system. Finally they outline two advanced projects concerned with creating a sculpture-based augmented reality that allows for the interactive participation of the observer.

Since the inception of computing, the tantalizing possibility that one day computers will be able to produce artifacts automatically, without human intervention, has been advanced by a number of artists, scientists and science-fiction writers. The human desire to communicate with other sentient beings, even beings of our own creation, that exhibit an alien type of intelligence and creativity has fueled research in artificial intelligence and remains a fascinating research question. More realistically, however, the use of computer hardware and software to aid the artist in the creative process can be seen as a set of sophisticated tools, analogous to the use of the brush and canvas or the chisel and marble, leading to machine-mediated augmentation of human creativity.

In this paper we focus our attention on the area of computer-aided sculpture and carving, a human activity aimed at producing new shapes by cutting or engraving some initial lump of material. Currently, the computer can aid the full activity cycle from sketching the initial idea of a sculpture and exploring a variety of different shapes all the way to the physical production of the final sculpture. Existing methods and tools, as well as known works in computer-aided sculpture and carving, are discussed in the following section.

We then present a new approach: Starting with a physical sculpture, we produce its computer model and manipulate this model to produce new shapes that can eventually be manufactured to produce a new physical sculpture. We call this approach "augmented sculpting," as it extends the existence of physical artifacts to a virtual computer world and then closes the loop by bringing new computer models into physical existence.

The technology on which this work is based is known as the Function Representation (FRep) of geometric models. The original mathematics, algorithms and tools that we use have been developed and continue to be developed by the HyperFun project team. HyperFun is an international free software project on functionally based shape modeling, visualization and animation <<http://www.hyperfun.org>>. Members of

HyperFun Team -- a freely associated group of researchers and students from different countries -- have contributed to the augmented sculpture study presented here.

Related Works

Currently, many diverse developments are taking place in digital sculpture [1]. There are two main concerns in the creation of the computer model of a sculpture: the way we model its shape and the way we represent this model.

Shape design can be performed in various ways, using either a purely mathematical description or an interactive modeling technique employing pressure-sensitive or haptic interactions and virtual-reality tools (i.e. a cyberglove and a head-mounted display).

From the technological point of view, tools for computer-aided sculpting have been described as "a system that will allow the reshaping of illusory computer material by carving into it, adding material to it or modelling it [to] open new and exciting creative opportunities" [2]. More recently, sculpting techniques providing global and local deformations have become one of the central themes in computer-graphics literature [3--5]. Another approach is based on interactive local modifications of a basic shape using a carving tool modeled on a chisel, set-theoretic operations, offsetting and blending. This approach has been implemented in volumetric (voxel) models [6,7], constructive solid geometry (CSG) [8] and functionally based models [9]. The production of evolutionary 3D aesthetically pleasing shapes is also a promising direction for computer-aided sculpting [10]. The presentation of the modeling results can be done by hand-manufacturing the final sculpture from physical materials, by utilizing rapid-prototyping machinery [11] to produce physical objects, or in the form of still images, animation or virtual-reality presentations.

Let us illustrate the variety of styles in computer sculpture using works of well-known computer researchers and artists in this field. Carlo Sequin is a computer scientist who experiments with modeling unusual shapes using parametric and subdivision surfaces. He produces these shapes in wood or bronze, either by hand or using stereolithographical rapid prototyping. He also works closely with Brent Collins, a professional wood sculptor who produces physical versions of computer-designed shapes by hand-cutting slices from wooden boards, laminating and finely polishing the sculpture. Sequin stresses that a computer program becomes an amplifier of one's creativity in this process [12].

Helaman Ferguson sees himself as both a mathematician and a sculptor. He borrows ideas from mathematics to produce stone sculptures. He states: "I use many different kinds of specialized tools, including computers, virtual image projection from equations, tool position and orientation monitoring, air hammers and drills, carbide cutters". His computer tool position and orientation monitoring system does not do the cutting work, it just provides quantitative information used during the hand cutting [13].

Bruce Beasley is a sculptor who has been using computer-aided design tools since the early 1980s. The computer helps him to experiment with 3D polygonal shapes and to transfer the elements of the model finally selected to foam core (polystyrene sandwiched between paper) using a plotter. The foam core components are used as patterns to guide the joining of bronze plates into a sculpture [14].

In the early 1990s, Mike King, working at the intersection of art and science, implemented a 3D modeling software system called Sculptor [15]. His system supports both user-guided creation of arbitrary primitive objects (composed of overlapping spheres) and algorithmic (procedural) object evolution. He has produced some of the modeled objects as physical sculptures using a contour-slicing routine and plywood laminae.

Stewart Dickson is both an artist and a computer specialist. He experiments with various mathematical and computer graphics software and realizes the computed shapes using stereolithography and other rapid prototyping equipment [16]. In his recent works, he has used a 3D color printer to produce volumetric objects with internal color distribution.

William Latham is an artist who, in collaboration with computer scientist Stephen Todd, worked on creating complex and visually striking virtual shapes employing methods and themes of natural evolution and artificial life. Latham and Todd have implemented a suite of interactive tools encompassing the entire computer-based process for the creation of evolutionary art forms, starting from the initial concept and hand-drawn sketches, through the definition and application of their distinctive artistic "evolutionary" style, to the rendering of 3D forms and the creation of gallery artworks and animations [17].

Alexei Sourin is a computer scientist interested in various craft forms, such as carving, embossing and woodcutting. He has implemented an interactive system for virtual copper-plate embossing and realistic rendering of the computed results [18]. A pressure-sensitive tablet is used for deeper immersion of the user and closer imitation of the real embossing process. He tested the system by producing virtual replicas of his earlier handmade works. It is worth noting that he uses a function representation of geometric shapes, which is also used in our work and is described in the following section.

Functionally Based Methods and Tools

In this section, we provide a brief introduction to the functionally based methods and tools that form a technical base for implementing our augmented sculpture study. Further technical information relating to these methods can be found in the glossary.

Function Representation and Hypervolumes

The idea of representing artworks with analytical mathematical functions can appear rather strange and impractical to readers from the art world. Our approach is to provide shape-modeling tools for computer art and is based on the FRep of the underlying model in a unified manner [19]. This modeling approach makes it possible to represent by a single continuous function such diverse objects as traditional skeleton-based implicit surfaces, convolution surfaces, constructive solids, swept objects and volumetric objects.

Recently, a more general “Hypervolume” model has been introduced that allows for modeling multidimensional point sets with attributes [20]. A point set is an FRep-based geometric model of a real object. An attribute is a mathematical model of an object property of an arbitrary nature (material, photometric, physical, etc.). This modeling approach is general enough to cover objects in solid modeling, heterogeneous object modeling and volumetric graphics.

The HyperFun Language

HyperFun [21] is a specialized high-level language that allows for a parameterized description of functionally based multidimensional geometric shapes. While being minimalist in design and easy to master, it supports all the main concepts of FRep. This language was designed to be as simple as possible in order to allow non-specialist users to create models of complex geometric shapes.

A model in HyperFun can be constructed using assignment statements, conditional selection statements and iterative statements. Functional expressions are composed using conventional arithmetic and relational operators. It is possible to use standard mathematical functions as well as special built-in operators, which support fundamental set-theoretic operations.

In principle, the language is self-contained and allows users to build objects from scratch, without the use of any pre-defined primitives. However, its expressive power is greatly increased by the availability of the system “FRep library,” which is easily extendable and can be adapted to particular application domains and even customized to the needs of particular users. Currently, the version of the FRep library in general use contains the most common primitives and transformations of a quite broad spectrum.

The HyperFun Software Tools

The following three software tools are available for download free of charge from the HyperFun Project web site <www.hyperfun.org>.

- *The HyperFun Polygonizer* (HFP). This program polygonizes and displays an object input from a HyperFun file. It has a command-line interface allowing the user to define a number of options and to specify such rendering parameters as “DisplayMode,” “GridDensity,” “BoundingBox,” the “FaceColor,” the “LineColor,” etc. Support for higher dimensional models is also available. For Web viewing, the program also makes it possible to output the results in VRML format.
- *The HyperFun for POVray* (HFpov) is a plug-in to the popular ray-tracer POVray <www.povray.org>, which makes it possible to generate high-quality photorealistic images on an ordinary PC. HyperFun objects can be manipulated as POVray objects. All ray-tracing options can be set using POVray scene descriptions. Animation capabilities are also available.
- *HyperFun for Windows* (HFW) is an interactive system allowing the user to easily master FRep modeling concepts using the HyperFun language while working in a conventional Microsoft Windows environment. This program has an interactive windows graphical user interface with pop-up menus and

toolbars. It allows the user to specify an FRep model (using a built-in text editor) in the HyperFun language, to compose complex scenes with multiple objects, to specify visual parameters for subsequent rendering and to generate animated sequences. This tool has been used for student exercises in a number of computer-graphics university courses in Japan and Russia.

The Augmented Sculpture Case Study

In this section, we present a detailed description of a sculpture-based case study, which allows us to demonstrate the use of a variety of functionally based modeling techniques available to an artist using the HyperFun toolkit.

Modern abstract sculpture is very much concerned with the exploration of novel, non-trivial shapes, and we believe that the computer technology being presented here can assist artists in this creative quest. Functional representation methods actually employ constructivist techniques that allow the emulation of physical or virtual “building blocks” in the form of primitive geometric shapes that can be combined in complex spatial relationships. A computer-based means of sculptural representation joined with a specific virtual environment in which the sculptural shapes are set can lead to the production of artifacts presenting a new aesthetic. Consequently, viewers experiencing these shapes within a virtual space can also benefit from this technology. Note that modern rapid-prototyping machines [22] allow the manufacture of any of the virtual sculptures presented in this paper, using real materials (such as paper or starch-based or plaster-based powder).

Our case study is based on real (physical) sculptures created by Russian artist Igor Seleznev [23]. He became the artist in residence for the HyperFun project in 1999. Since then, he has collaborated with HyperFun team on a regular basis.

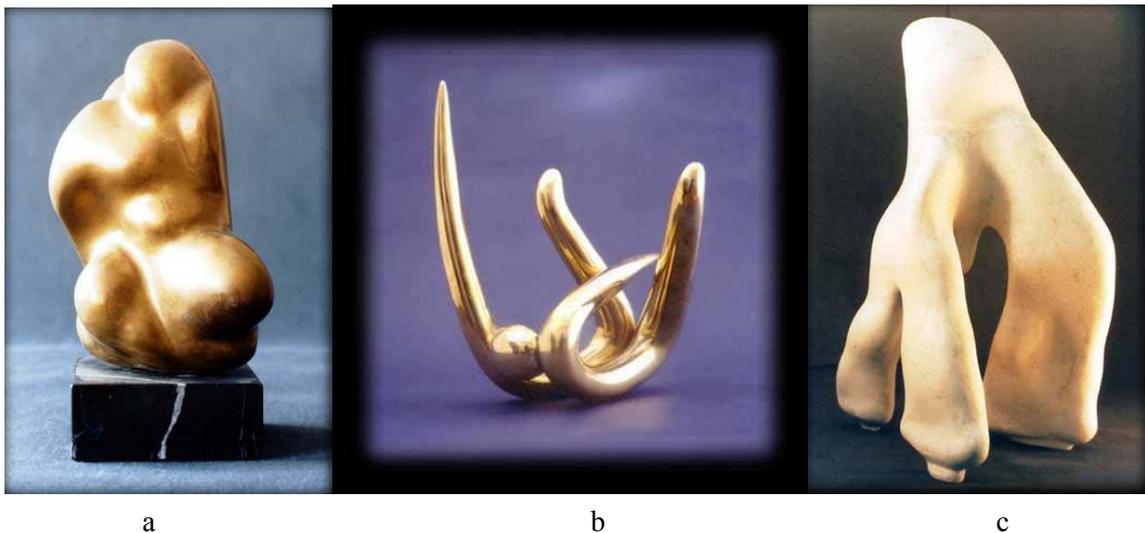


Fig. 1: The physical sculptures (a) *Naked* {(1993)}, bronze statuette, 28 cm in height; (b) *Gymnast or Plasticity 3* {(1990)}, bronze statuette, 22 cm in height; (c) *Walking Androgynous* {(1995)}, marble statue 110 cm in height. (© Igor Seleznev)

Real Sculptures and Their Computer Models

In our discussion, we will examine the modeling of three “real” sculptures (*Naked*, *Gymnast*, *Walking Androgynous*) illustrated in Fig. 1. First, it was necessary to construct the geometric models of those sculptures. All three selected sculptures have quite complex shapes with subtle features. Modern mathematics provides a variety of functions for modeling very complex non-regular forms. In particular, so-called convolution surfaces [24,25] (see glossary) are very attractive in this respect. The FRep library contains a number of predefined functions that only require the user to specify a number of intuitively clear parameters, thus greatly facilitating the modeling process without requiring the user to have a good understanding of the underlying mathematics.

The organizational aspect of the modeling process was noteworthy in its own right. The creation of the geometric models of the three sculptures was undertaken as an exercise by computer-graphics students of the Department of Computer Science at the Moscow Engineering Physics Institute (MEPhI). Initially,

photographs of the sculptures were made available to the students via the Web. The students were encouraged to write collaboratively and share the HyperFun code to create the rough prototype geometric models. The students then formed groups of two or three to combine their efforts in building more accurate models. The physical sculptures were made available to the students at this stage, and the artist himself took part in assessing and discussing the intermediate results with the students, both in person and through the Internet. Finally, the students constructed geometric models of the sculptures in the form of a HyperFun language program, as well as a set of photorealistic renderings of the sculptures using the same program (see Fig. 2). Model building and debugging was carried out using the HyperFun for Windows toolkit, and the authors generated the final ray-traced renderings using the HyperFun for POV-Ray toolkit.

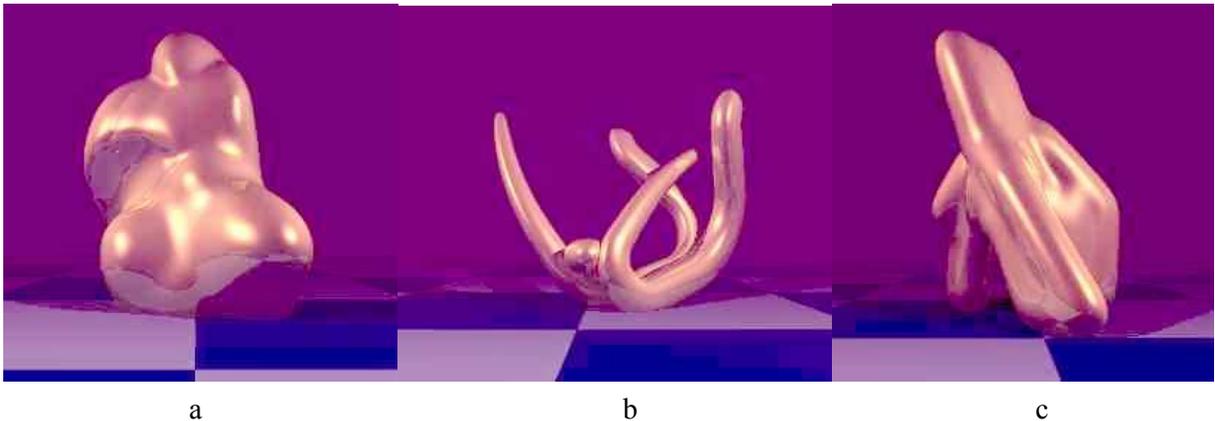


Fig. 2: The geometric models programmed by the students of Moscow Engineering Physics Institute (MEPhI). (a) *Naked* modeled by A. Dyakonov and E. Krivochenko, using the following FRep functions: ellipsoids (hfEllipsoid), convolution surfaces (hfConvPoint, hfConvLine), affine[CORRECT WORD?] transformations (hfShift3D, hfScale3D, hfRotate3D), blending intersection and union operations (hfBlendInt, hfBlendUni); (b) *Gymnast* modeled by P. Yablochkin and N. Varivoda. They defined the basic geometric primitives by formula and then used the following FRep functions: non-linear deformations (hfBending3D), affine transformations (hfShift3D, hfScale3D, hfRotate3D) and blending (hfBlendInt, hfBlendUni); (c) *Walking Androgynous* modeled by S. Berzin and S. Sustavov using the following FRep functions: convolution surfaces (hfConvLine) and blending union (hfBlendUni). (© V. Adzhiev)

Texturing the Geometric Models of the Sculptures

Of course, the geometric models of the sculptures are not in themselves of much interest, as they are simply fairly accurate representations of the physical sculptures. What is much more interesting is that once modeled geometrically, they can be used as “base shapes” to help the artist explore the conception of new artifacts that would be impossible to create without the use of the computer.

Probably the most obvious variation would be to change the material from which the sculpture would be constructed. Artists normally have a limited set of materials available, dictated both by aesthetic and technological factors. Modern computer graphics, on the other hand, provides the artist with an almost limitless set of “virtual materials.” In traditional sculpture, using a new material can be very problematic (one may recall, for instance, how hard Bruce Beasley had to work to introduce the acrylic casting process that allowed him to make transparent sculptures). On the other hand, using computer graphics makes it relatively easy to render a glass sculpture (see *Naked* in Fig. 3a).

Much more interesting results can be achieved using different types of texturing. For example, let us consider the process of applying a constructive solid texturing technique to *Naked*. A novel texturing method based on the Hypervolume modeling technique, briefly described above under "Function Representation and Hypervolumes," has been used. For every point inside a shape, one can assign a unique set of optical attributes. This approach facilitates the creation of optically refined Lucite sculptures. In particular, one can place light and space inside a sculpture to imbue it with “inner life,” which results in impressive visual ambiguity.

On a more technical level, to generate a sculpture made of glass with multicolored spiral striations going through the material, we proceed as follows. First, let us assume that we have a clear block of glass from which the sculpture, shown in Fig. 3a, is made. Then we create the solid model of three interlocked

spirals shown in Fig. 3b. Finally, we assign different shading parameters to these spirals and compute the union of the clear glass block and the spirals. This new material was used to create the striped class sculpture of Fig. 3c.

This Hypervolume modeling approach can also allow the implementation of much more complex effects. For instance, one can design the material block from which a sculpture is carved using the geometric model of a second sculpture. The possibilities for experimentation become endless.

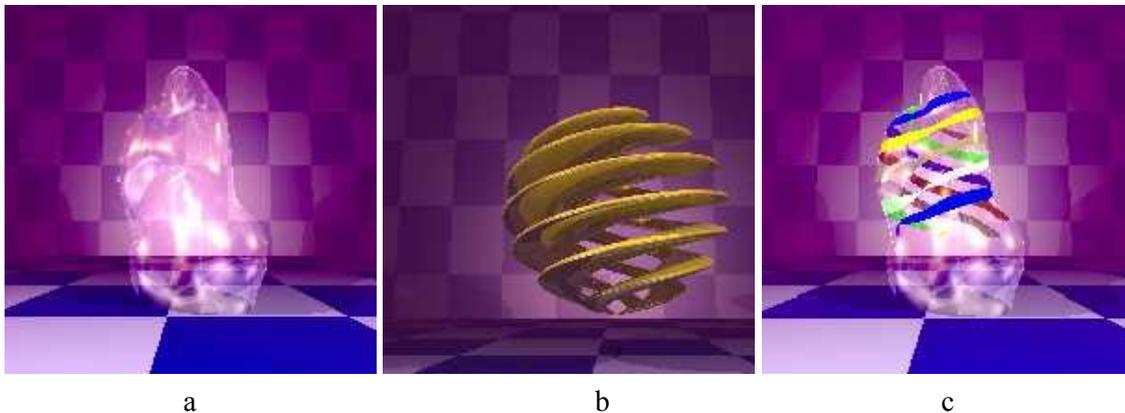


Fig. 3: Constructive solid texturing; models implemented by Benjamin Schmitt (University of Bordeaux, France), a member of HyperFun Team: (a) the original model of *Naked* as a glass sculpture, (b) a constructive 3D solid object representing space partition, (c) the textured shape. (© V. Adzhiev)

The Animation of Sculptures

What would be quite interesting, and obviously impossible without computer technology, would be to have the ability to animate a sculpture that, while being initially still, has internal (although hidden) dynamics giving it a natural propensity to move.

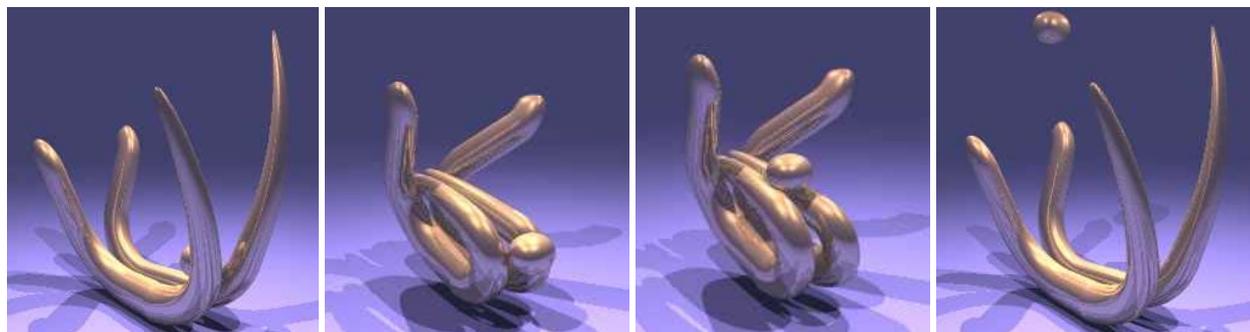


Fig. 4: Four frames of *Gymnast* animation sequence; modeled by MEPHI student Alexander Ogarko using the “HyperFun for [POVRay]” toolkit. The entire animation can be played at <http://wwwcis.k.hosei.ac.jp/~F-rep/App/ASP/FASP.html>. (© V. Adzhiev)

There is a traditional division in computer animation between the modeling and animation processes, which leads to limitations on the range of forms that can be animated. The advantage of the FRep is that one can work with both the shape and its animation simultaneously. Thus time-dependent shapes can be modeled directly as four-dimensional (4D) objects with the subsequent generation of time cross-sections (3D frames) taken at discrete values along the time dimension. So, a 4D “space-time” FRep model defined by the inequality $F(x,y,z,t) \geq 0$ provides a natural solution to this problem.

Among the three sculptures that we have examined so far, *Gymnast* appears to be the most suitable candidate for animation. The geometric model of this sculpture has been redeveloped and 10 parameters (bending angles for individual joints, etc.) intended to aid the animation process were introduced. Figure 4 shows a few frames from the animation sequence produced.

Multidimensional Modeling for Shape Metamorphosis

Four-dimensional “space-time” modeling is just a particular case of multidimensional, functionally based modeling that allows us to accomplish non-trivial effects while modeling and animating. One important application of this type of modeling, which is of great interest to both artistic and commercial

animation and even to medical applications, is to the problem of shape metamorphosis (sometimes known as 3D morphing). In the FRep framework, metamorphosis is performed almost trivially [26] (see glossary), and the HyperFun toolkit supports an original technique that allows us successfully to solve the metamorphosis problem by using FReps to represent the key-shapes.

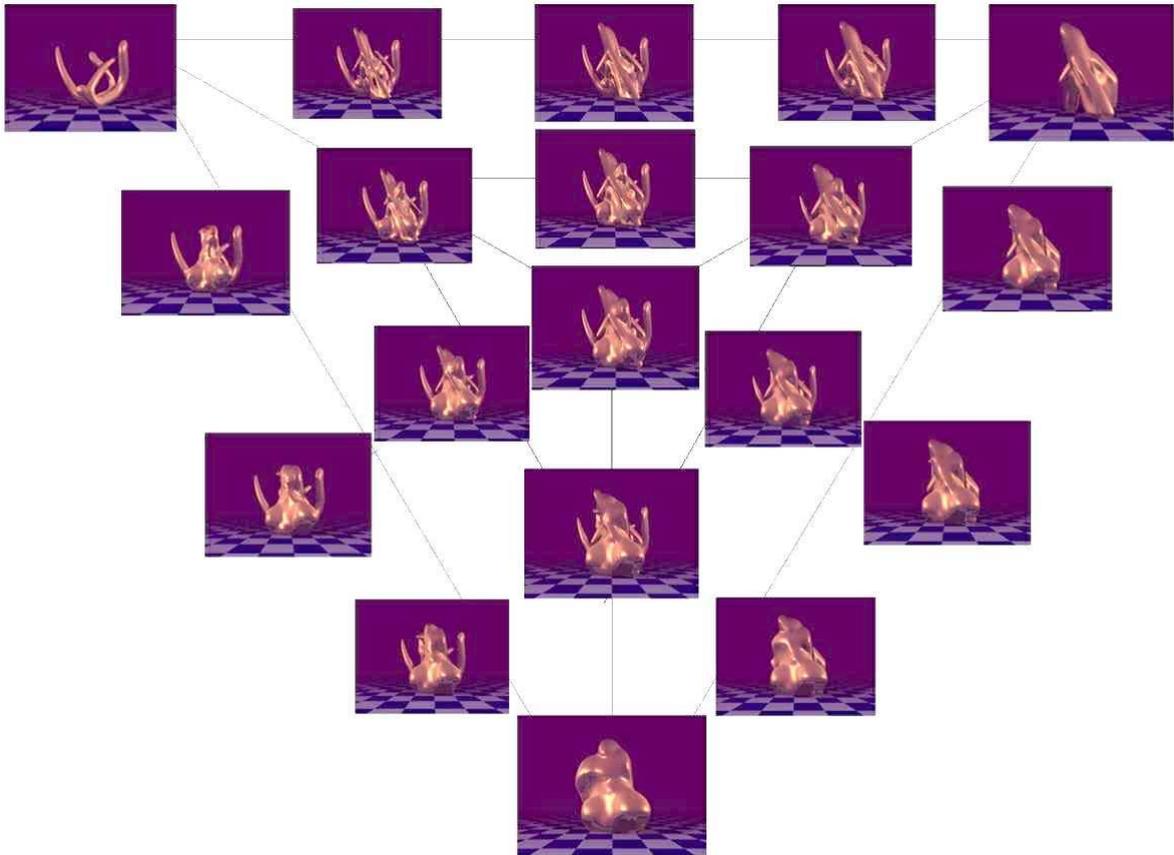


Fig. 5: Metamorphosis “triangle spreadsheet”; modeled by MEPHI student Alexander Ogarko. (© V. Adzhiev)

To perform a non-trivial metamorphosis where more than two objects are involved, we have introduced special “spreadsheet coordinates.” This spreadsheet-like coordinate system requires us to organize a set of 1D, 2D or 3D nodes, consisting of elementary images or shapes, in some sort of tabular arrangement. For example, a rectangular arrangement of these nodes gives rise to a “rectangular spreadsheet” metamorphosis scheme, which in mathematical terms is defined as a bilinear interpolation of the four key-shapes situated at the corners of the rectangle [27].

A similar approach was used to handle the three sculptures discussed above. Having generated the 3D geometric models of the three sculptures shown in Fig. 1, the dimensionality of an integral “metamorphosis model” was increased to 5D to include two additional coordinates, x_4 and x_5 , which were mapped onto the polar coordinates (r, α) . As we wished to produce new shapes by interpolating the three initial key-shapes of the sculptures, we introduced a “triangular spreadsheet” interpolation scheme that utilized the polar coordinates. In this arrangement, the corners of the triangular spreadsheet are the initial key-shapes of the sculptures. Thus, an intermediate shape of the metamorphosis at any given point inside the triangle is represented by the weighted sum of the defining functions of the initial key-shapes.

Figure 5 shows a number of intermediate shapes at different points inside the triangle, produced by metamorphosing the three initial key-shapes. Each image was produced using direct ray-tracing of the shape defined by the HyperFun model using the HyperFun for POV-Ray toolkit. Figure 6 represents the interpolated shape at the center of the triangle. The most important aspect of this approach is that the artist can use it to discover novel shapes starting from a series of key-shapes. The artist may have only a vague idea of the shapes sought after and may discover quite unexpected but interesting shapes by progressive refinement. A series of metamorphosed shapes generated in this manner were assessed by our resident artist, who was quite impressed by their unusual forms and even declared his intention to mould some of them.



Fig. 6: The interpolated shape in the center of the metamorphosis triangle is in a sense the "average" derived from metamorphosing *Naked*, *Gymnast* and *Walking Androgynous*. (© V. Adzhiev)

Next we determined to produce an animation sequence depicting the gradual metamorphosis between the initial key shapes of the sculptures. First, we mapped the abstract coordinates to multimedia coordinates: $x_4 \rightarrow t_1$, $x_5 \rightarrow t_2$. Then we defined a spiral animation path on the t_1t_2 plane, which successively passes through the points labelled 1 through to 21 in Fig. 7. The resulting animation is available at <<http://wwwcis.k.hosei.ac.jp/~F-rep/App/ASP/FASP.html>>.

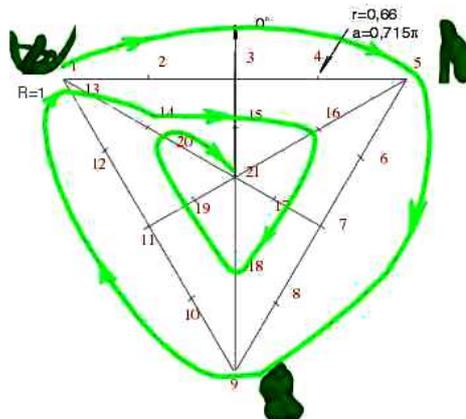


Fig. 7: A spiral animation path on the t_1t_2 plane. The path passes through the points labeled 1 through 21. The resulting animation sequence of gradual metamorphosis between three sculptures situated at the corners of the triangle is available at <<http://wwwcis.k.hosei.ac.jp/~F-rep/App/ASP/FASP.html>>. (© V. Adzhiev)

Projects in Progress

In this section we briefly outline two additional projects, which are currently under development.

Navigation and Sculpting in an FRep Sculpture Garden

The next stage of development of our computer-mediated-sculpture study was perhaps the most consequential one yet, namely the development of a system that allows the artist to experience real-time interaction with a virtual sculpture. The description of a similar project, called the Genetic Sculpture Park, appeared recently in *Leonardo* [28].

HyperFun team member Maxim Kazakov, of the Moscow Institute of Physics and Technology, has developed and is in the process of refining an interactive system based on an original technique [29]. This system allows the user to navigate interactively through a so-called Augmented FRep Garden, which is a time-dependent scene composed of multiple objects. In this virtual environment it is possible for the user to interactively edit, on the fly, the objects populating this environment as well as the processes that metamorphose these objects over time.

The three by-now-familiar sculptures are placed at the vertices of an equilateral metamorphosis triangle, the center of which acts as a placeholder for the result of the metamorphosis. Here we use the same geometric models for the initial and the metamorphosed shapes, but a different method for the visualization of the resulting scene that allows for rendering in real time.

Let us describe how the user interacts with the model during the sculpting process.

- The user is able to navigate the initial scene, selecting an appropriate point of view.
- Once the point of view has been chosen, the user can begin the process of metamorphosis. Using a special virtual tool, the user can select a point inside the triangle and thus determine the corresponding resulting shape. This is an iterative process, which allows the user to arrive at an aesthetically pleasing shape.
- Once the desired shape is arrived at, the user can further refine it using a carving tool. This tool can be immersed into the surface of the shape, allowing the user to both remove material from and add material to the shape. It is even possible to design a new shape ab initio using this tool.
- The results of the carving process can be discarded by using the metamorphosis tool to select some other point in the metamorphosis triangle.

Our initial experience with this system shows that it is easy to master, even for children. The system is efficient enough to maintain interactivity with all operations, even on an ordinary personal computer with an average graphics card.



Fig. 8: A rough sketch of the Augmented Sculpture installation. (© Igor Seleznev)

The Augmented Sculpture Installation

In this section we outline a proposal for an interactive art installation. The title of the exhibit will be *Augmented Sculpture*, denoting a form of sculpture that is both physical and virtual, in which the virtual world is overlaid on the physical world. The physical aspects of this installation will include the physical sculptures, produced by the HyperFun resident sculptor Igor Seleznev, and the space in which this installation is exhibited. The virtual aspects of this installation will include a series of computer-generated images and animations depicting the sculpting process, which will be displayed and projected on various monitors and surfaces of the exhibit.

All the technological problems associated with creating such an exhibit have already been resolved, as seen in previous sections of this paper. The physical sculptures of our resident sculptor will constitute the key-shapes that will give rise to the virtual artifacts produced by the interactive sculpting sessions. We intend the viewers of the exhibit to be active participants in the sculpting process, interacting with the exhibit and producing their own virtual sculptures in real time. Some computer-generated views and animations will be dependent on the position and orientation of the viewer inside the exhibit. Thus, the viewer will be able to experience and affect the virtual part of the exhibit. A CD-ROM with information on the installation (including the geometric models, the animation sequences, the HyperFun tools and tutorials) may be created and distributed to the public.

Depending on the environment, the involved media, the installation dynamics and the degree of user immersion, a number of alternative installations are possible.

- A rough sketch of the basic installation is shown in Fig. 8. The sketch shows a triangular pyramid, at the vertices of which are situated four physical sculptures that will act as the key-shapes of the interpolation process. The sculptures *Walking Androgynous*, *Naked* and *Don Quixote* form the base of this pyramid and the sculpture *Gymnast* is suspended from the ceiling and lies at the apex of the pyramid. At the

middle of the triangular base of the pyramid lies a fifth sculpture that is the result of the interpolation the three sculptures at the corners of the base. At the centre of each side face of the pyramid lies a flat screen which is suspended from the ceiling. Each screen shows still images and animations of the shape resulting from the interpolation of the three key-shapes at the vertices of the corresponding pyramid face. At the center of the pyramid is a cluster of three monitors suspended from the ceiling and aligned with the side faces of the pyramid. These screens show still images and animations of the shape resulting from the interpolation of the four key-shapes at the vertices of the pyramid.

- Elements of temporal behavior of various kinds can be added to the basic installation. For instance, a dynamic effect can be produced by allowing the monitors to move inside the pyramid and causing the metamorphosis shape, shown on the monitor, to correspond to the monitor's location in space.
- Better viewer immersion can be achieved by allowing the viewer to move the flat screens along the pyramid faces using levers and handles. As the viewer moves the flat screen, it displays the gradually changing shape that corresponds to its position in the triangular face.
- Each viewer could be given a head-mounted display or a helmet with a semi-transparent screen used to project an animation corresponding to the current viewing position inside the pyramid. Such an arrangement would cause the viewer to be truly immersed into the augmented reality.
- A number of computers running the interactive navigation and sculpting system could be made available to members of the public, to allow them to experience for themselves the creation of a virtual sculpture.
- Alternatively one may use a rapid prototyping machine to manufacture a physical version of the created virtual sculpture. In particular, modern 3D printers [30] can be utilized to give viewers an opportunity to experience the complete cycle of virtual production.

Conclusion

In conclusion we would like to make the following observations.

The function representation of shapes that we have employed presents the system user with the opportunity to envision, discover and create novel sculptural forms without imposing many restrictions. This type of modeling, combined with evolutionary algorithms and virtual reality interfaces, points to a wealth of research possibilities.

Our initial, but as yet incomplete, study of the creative cycle from physical artifact to computer model and back to real sculpture through the use of computer-aided manufacturing or rapid prototyping methods constitutes the basis for a novel sculpting technique. Augmenting this form of sculpture with animation leads to a new art form that enables the experimental discovery and realization of computer-inspired and -mediated physical sculptures. We are planning to experiment with modern rapid prototyping machines to produce the virtual sculptures in real material such as plastic (via the SLA3500 stereolithography machine) and paper (using the KIRA Solid Center).

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Glossary

Constructive Solid Geometry (CSG) -- a solid modeling technique and an internal model representation based on logical (Boolean) combinations such as intersection, union and difference of predefined solid primitives (ball, cube, cone, cylinder, torus, etc.).

convolution surfaces -- a class of implicit surfaces allowing the modeling of complex free-form shapes with non-regular features. These surfaces look like smooth shapes swept by a ball moving along a skeleton composed of points, lines, arcs and triangles. Their definition is based on the integration of the kernel function along the skeleton.

Function Representation (FRep) -- a generalization of traditional implicit surfaces, CSG, sweeping and other geometric models. Mathematically speaking, an FRep defines a geometric object by the inequality $F(X) \geq 0$, where $X = (x_1, x_2, \dots, x_n)$ is a vector in n-dimensional Euclidean space. This approach makes it possible to represent, by a single continuous function, such diverse objects as traditional skeleton-based implicit surfaces, convolution surfaces, constructive solids (using the so-called R-functions for set-theoretic operations), swept objects and volumetric objects. The advantage of this modeling approach is that the result of any supported operation can be treated as the input for a subsequent operation; thus very complex models can be created in this way from a single functional expression. Accordingly, in an FRep there is no difference between soft objects, CSG solids and volumetric (voxel) objects, which are processed in the same manner. This allows us to solve such long-standing problems as metamorphosis between objects of different topologies, sweeps generated by a moving solid, controlled blending for all types of set-theoretic operations, collision detection and hypertexturing for arbitrary solids, direct modeling of space-time, multidimensional objects, etc.

FRep library -- a suite of pre-programmed procedures containing the most common primitives and transformations within a broad range[OK?]. Thus, there are functions implementing conventional CSG primitives (e.g. block, sphere, cylinder, cone and torus) as well as their more general counterparts (i.e. ellipsoid, superellipsoid, elliptical cylinder, elliptical cone). Another group of library primitives implements popular implicit surfaces (e.g. blobby objects, soft objects, metaballs) including the recently implemented convolution objects with skeletons of different types (i.e. points, line segments, arcs, triangles, curves and meshes). Primitives derived from parametric functions (i.e. cubic spline and Bézier objects) have also been implemented recently. The usual primitive transformation operations are available, such as rotation, scaling, translation, twisting, stretching, tapering, blending union/intersection, as well as some more general operations such as non-linear space mapping driven by arbitrary control points.

hypervolume -- a model allowing for modeling of multidimensional point sets with attributes. Point set geometry and attributes have an independent representation but are treated uniformly. A point set in a geometric space of an arbitrary dimension is an Frep-based geometric model of a real object. An attribute that is also represented by a real-valued function (not necessarily continuous) is a mathematical model of a real object, process or property of an arbitrary nature (material, photometric, physical, etc.).

implicit surface -- a surface in 3D space, defined by the set of points possessing an equal value for a given continuous function of three variables (other terms: iso-surface, zero-level set, equipotential surface).

metamorphosis (3D morphing) -- a process concerned with generating intermediate shapes between several "key-shapes," which traditionally requires the animator to establish correspondences between sets of points of the initial and final key-shapes of a metamorphosing object. This process is quite cumbersome and is incapable of dealing with key-shapes of differing topologies. In addition, using the traditional approach, it is difficult to obtain intermediate shapes by interpolating more than two key-shapes. In the FRep framework, metamorphosis is performed almost trivially using function interpolation. It is capable of handling key-shapes of differing topology and of generating intermediate shapes that exhibit genus change that may be composed of disjoint components.

multimedia coordinates -- in the FRep framework, a multidimensional object defined in abstract n-dimensional geometric space is given by an inequality of the form $F(x_1, x_2, \dots, x_n) \geq 0$. In parallel with this

“abstract world,” there exists another “real world” populated by “multimedia objects.” A multimedia object can be seen as a multidimensional object with a set of 2D or 3D geometric coordinates (e.g. Cartesian, cylindrical, etc.), a set of dynamic coordinates (e.g. physical time), a set of photometric coordinates (e.g. color, transparency, texture, etc.) and a set of audio or other “multimedia coordinates.” Both “abstract coordinates” and “multimedia coordinates” may assume values within given intervals.

The essence of our approach lies in introducing a space mapping between geometric coordinates and multimedia coordinates that effectively establishes a correspondence between an abstract multidimensional shape object and a real multimedia object. The HyperFun toolkit supports such a mapping with a variety of multimedia types. The “HyperFun for Windows” interactive system provides the user with a convenient way to define such a mapping through an interactive interface.

parametric surface -- a surface in 3D space whose geometry is defined by coordinates derived from two functions sharing some common parameters.

Subdivision surface -- a surface in 3D space defined by a polygon mesh and a set of refinement rules which recursively generate new meshes by refining the previous.

swept object -- a geometric object defined by the set of all points swept by 1D, 2D, or 3D object-generator along a trajectory through space. Such objects are especially useful in modeling the region swept out by a machine-tool cutting head or robot following a path.

texturing -- a special technique in computer graphics for mapping a given 2D image onto a 3D surface for simulation of colors, bumps and other surface properties in the rendering process. *Solid texturing* is characterized by the texture attributes definition for any point in 3D space thus avoiding the process of mapping 2D images onto a 3D-object surface. This means that when one applies a solid texture to an object, one has to create a space partition of the object space, where each subset contains a different material property.

volumetric modeling -- an area of shape modeling dealing with 3D objects in 3D space, in contrast to 2D surfaces and 1D curves. Homogeneous volumes (solids) can be modeled using a space partition into small cells (voxels) each having a binary occupancy index. Heterogeneous volumes have a vector of attributes assigned at each point and representing photometric (color, opacity, etc.), physical and other object properties.

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About authors

Valery Adzhiev is a research fellow in the National Centre for Computer Animation, Bournemouth University, UK. He received his MSc and PhD degrees in Computer Science from Moscow Engineering Physics Institute where from 1995 to 2000 he held a position of an Associate Professor. As a visiting researcher, he worked in the University of Warwick, UK (1993-94) and in the University of Aizu, Japan (1998-99). His research interests include solid and volume modeling, animation, agent-oriented modelling of complex systems and computer art. He has designed HyperFun modeling language and has contributed to implementation of HyperFun software tools.

Peter Comminos is the director of the National Centre for Computer Animation at Bournemouth University. He holds a BSc in Computer Science with Honours and a PhD in Computer Animation. For the past twenty-five years, he has taught widely on the mathematical, algorithmic and creative aspects of Computer Graphics, Geometric Modelling and Computer Animation. He has conducted research, has supervised research and has headed the Computer Animation research team of the University, which over the past ten years has been awarded the highest ranking in its field in the UK. He has designed and implemented the CGAL 3D Computer Animation system and language. He has published widely and has contributed to the production of numerous prize winning computer animated sequences.

Alexander Pasko is a professor in Hosei University, Japan. He received his MSc and PhD degrees in Computer Science from Moscow Engineering Physics Institute in 1983 and 1988. From 1993 to 1999 he worked at Aizu University, Japan. His research interests include solid and volume modeling, animation, multidimensional visualization, multimedia and computer art. He has developed a number of original techniques in functionally-based geometric modeling and has contributed to design and implementation of HyperFun software tools.