回路のインピーダンス

システムの周波数応答

Ｒ，Ｃ，Ｌ　の複素インピーダンス

- 抵抗: $R \equiv \tilde{Z}_R = Z_R$
- コンデンサー: $\frac{1}{i\omega C} = -\frac{i}{\omega C} \equiv \tilde{Z}_C$
- コイル:$i\omega L \equiv \tilde{Z}_L$

$$\tilde{Z} = \frac{\tilde{V}}{\tilde{I}}$$

- 単位:Ω

- 例:　50 Hz, C = 0.1μF, L = 1 μH

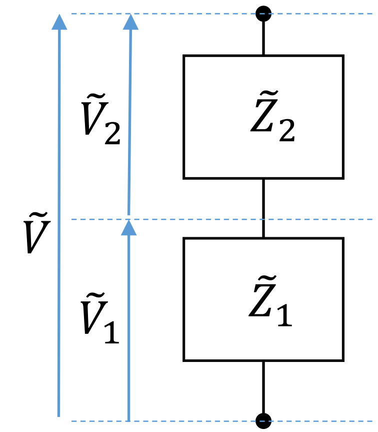
$$\omega = 2\pi \times 50 \text{ rad/s} \simeq 314 \text{ rad/s}$$

$$Z_C = \left|\tilde{Z}_C\right| \simeq 32 \text{ k}\Omega, Z_L = \left|\tilde{Z}_L\right| = 314\ \mu\Omega$$

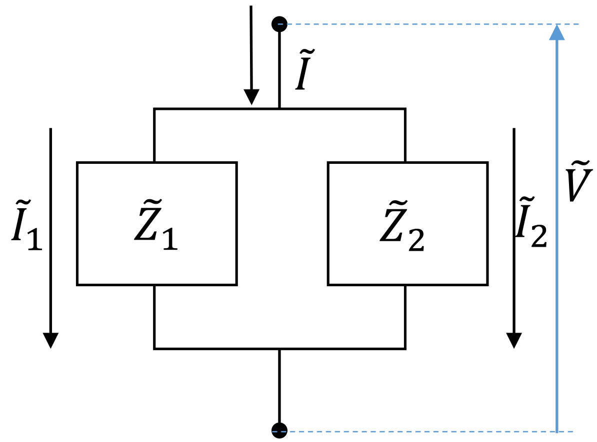
複素インピーダンスの合成

- ユニットの複素インピーダンスZ

  $\tilde{V} = \tilde{Z}\,\tilde{I}$ (複素振幅)



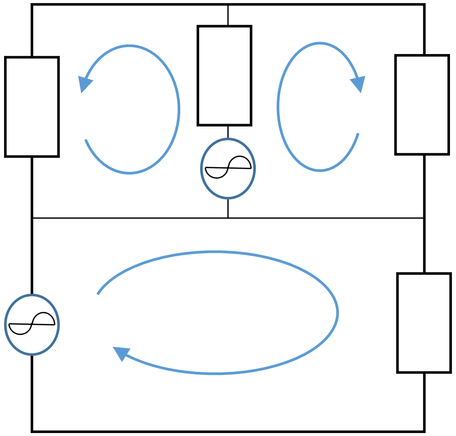
- 直列接続
  - 共通の電流 : $\tilde{I}$
  - 電圧の和: $\tilde{V} = \tilde{V}_1 + \tilde{V}_2 = (\tilde{Z}_1 + \tilde{Z}_2)\,\tilde{I}$
  - 合成インピーダンス : $\tilde{Z} = \tilde{Z}_1 + \tilde{Z}_2$



- 並列接続
  - 共通の電圧 : $\tilde{V}$
  - 電流の和 : $\tilde{I} = \tilde{I}_1 + \tilde{I}_2 = \left(\frac{1}{\tilde{Z}_1} + \frac{1}{\tilde{Z}_2}\right)\tilde{V}$
  - 合成インピーダンス : $\tilde{Z} = \frac{\tilde{Z}_1\tilde{Z}_2}{\tilde{Z}_1 + \tilde{Z}_2}$

LCR回路網

- 直流回路網の解析と同様
  - 抵抗　→　複素インピーダンス
  - 電源　→　交流電源
  - 電流　→　複素電流(振幅)
  - 電圧　→　複素電圧(振幅)

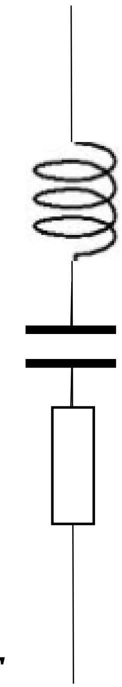


CLR 直列共振回路

- Zの周波数依存性

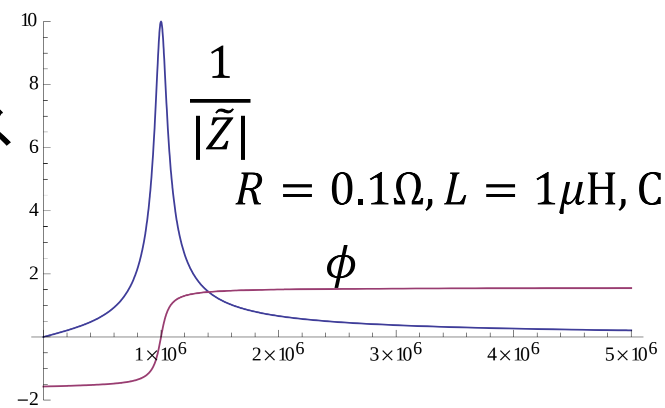
- $\tilde{Z} = R + \frac{1}{i\omega C} + i\omega L = R + i\left(\omega L - \frac{1}{\omega C}\right) = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\, e^{i\phi(\omega)}$

$$\phi(\omega) = \arctan\frac{\omega L - \frac{1}{\omega C}}{R}$$



$\omega = \frac{1}{\sqrt{LC}}$: 共鳴(共振)周波数,

純抵抗Rと同じ, 最小インピーダンス



$R = 0.1\Omega, L = 1\mu\mathrm{H}, \mathrm{C} = 1\mu F$

Q 4.1

- LCR並列共振回路の複素インピーダンスを計算し，その周波数依存性を論じよ．

A 4.1

- $\tilde{Z} = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C\right)^{-1} = \left(\frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)\right)^{-1}$

$$= \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} e^{i\phi} = \frac{R}{\sqrt{1 + \frac{R^2}{\omega^2 L^2}\left(1 - \omega^2 CL\right)^2}} e^{i\phi},$$

$$\tan\phi = -\frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}} = \frac{R}{\omega L}\left(1 - \omega^2 CL\right)$$

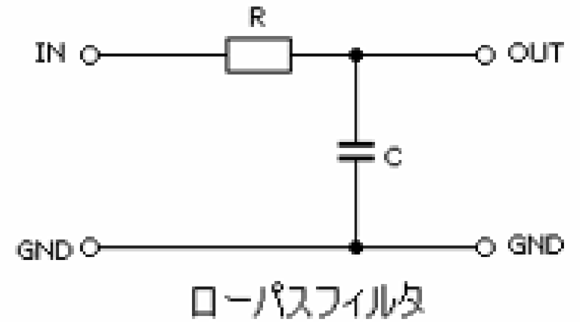
フィルター回路

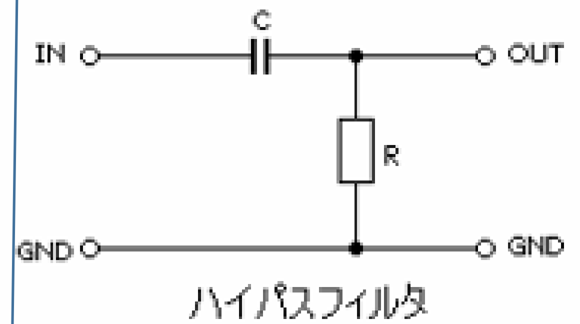
- $\tilde{V}_{in} = \left(R + \frac{1}{i\omega C}\right)\tilde{I}, \tilde{V}_{out} = \frac{1}{i\omega C}\tilde{I}$

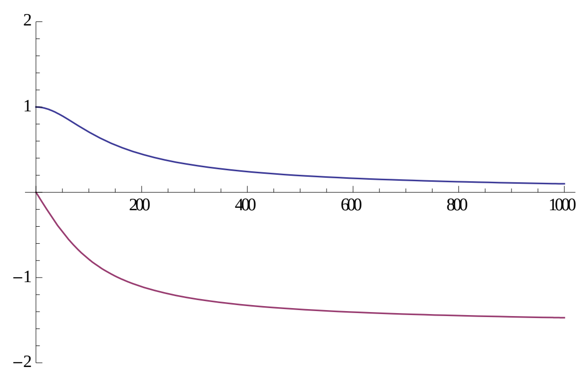
$$\tilde{V}_{out} = \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}}\tilde{V}_{in} = \frac{\tilde{V}_{in}}{i\omega RC + 1},$$

$\tau \equiv RC$

$$\frac{1}{i\omega RC + 1} = \frac{1}{\sqrt{1 + \omega^2\tau^2}}e^{i\phi}, \phi = \arctan(-\omega\tau)$$







Q 4.2

- RC high-pass filter について $\frac{\tilde{V}_{out}}{\tilde{V}_{in}}$ を計算せよ

交流電力

$$< P > = \frac{1}{T}\int_0^T P(t)\,dt = \frac{1}{2}\mathrm{Re}[\tilde{I}\,\bar{\tilde{V}}]$$

$$= \frac{1}{2}\,\mathrm{Re}\left[\tilde{Z}\,\tilde{I}\bar{\tilde{I}}\right] = \frac{1}{2}\,\mathrm{Re}\left[\tilde{Z}\,\left|\tilde{I}\right|^2\right] = \frac{1}{2}\,\mathrm{Re}[\tilde{Z}\,I_0^2]$$

$$= \frac{I_0^2}{2}\,\mathrm{Re}[\tilde{Z}]$$

Q 4.3

- LCR直列共振回路に供給される有効電力を求めよ