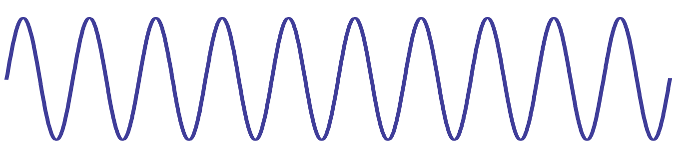
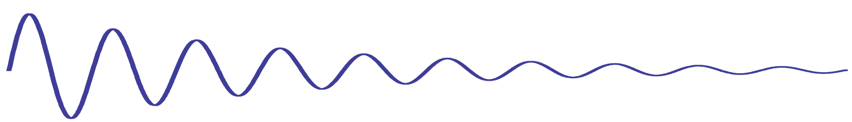
# 減衰振動
# 要点・復習

# 単振動　vs　減衰振動

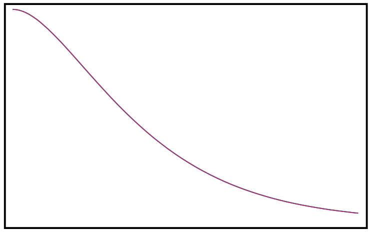
| | |
|---|---|
| **復元力**：$-kx$ | 復元力＋摩擦力: $-kx - c\,\frac{dx}{dt}$ |
| $x(t) = A\cos(\omega_0 t + \phi)$<br>$\omega_0 = \sqrt{\frac{k}{m}}$ | $x(t) = A\,e^{-\frac{\gamma}{2}t}\cos(\Omega t + \phi)$<br>$\Omega = \sqrt{\underbrace{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}_{\text{正のとき}}}$, $\gamma = \frac{c}{m}$ |
|  |  |
| 力学的エネルギー保存<br>$E = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\mathrm{max}}^2$ | 力学的エネルギー非保存<br>$E = E_0\,e^{-\gamma t}$ |

# 減衰振動の仲間

- 過減衰 $\frac{\gamma}{2} > \omega_0$

$$\boxed{x(t) = A\,e^{-\left(\frac{\gamma}{2}+\sqrt{\left(\frac{\gamma}{2}\right)^2-\omega_0^2}\right)t} + Be^{-\left(\frac{\gamma}{2}-\sqrt{\left(\frac{\gamma}{2}\right)^2-\omega_0^2}\right)t}}$$

$$A = \frac{2v_0 + \left(\gamma - 2\sqrt{\left(\frac{\gamma}{2}\right)^2 - \omega_0^2}\right)x_0}{4\sqrt{\left(\frac{\gamma}{2}\right)^2 - \omega_0^2}}, \qquad B = \frac{2v_0 + \left(\gamma + 2\sqrt{\left(\frac{\gamma}{2}\right)^2 - \omega_0^2}\right)x_0}{4\sqrt{\left(\frac{\gamma}{2}\right)^2 - \omega_0^2}}$$



- 臨界減衰 $\frac{\gamma}{2} = \omega_0$

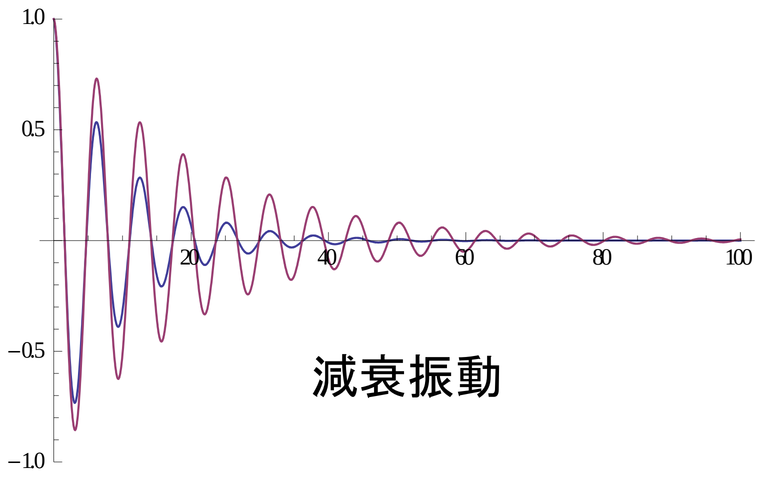
$$\boxed{x(t) = (at+b)e^{-\frac{\gamma}{2}t} = (at+b)e^{-\omega_0 t}}$$

$$a = v_0 + \left(\frac{\gamma}{2}\right)x_0, \qquad b = x_0$$

# 減衰振動の振動数

どのような振動数の単振動を重ねると，減衰する振動となるか？

- フーリエ変換

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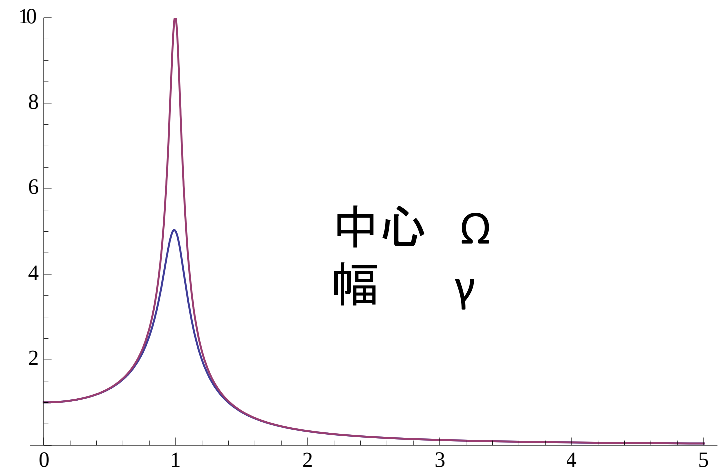


振幅のスペクトル