KMS-Cube

A General Alternative to Hypercubes for Reducing the Node Degree

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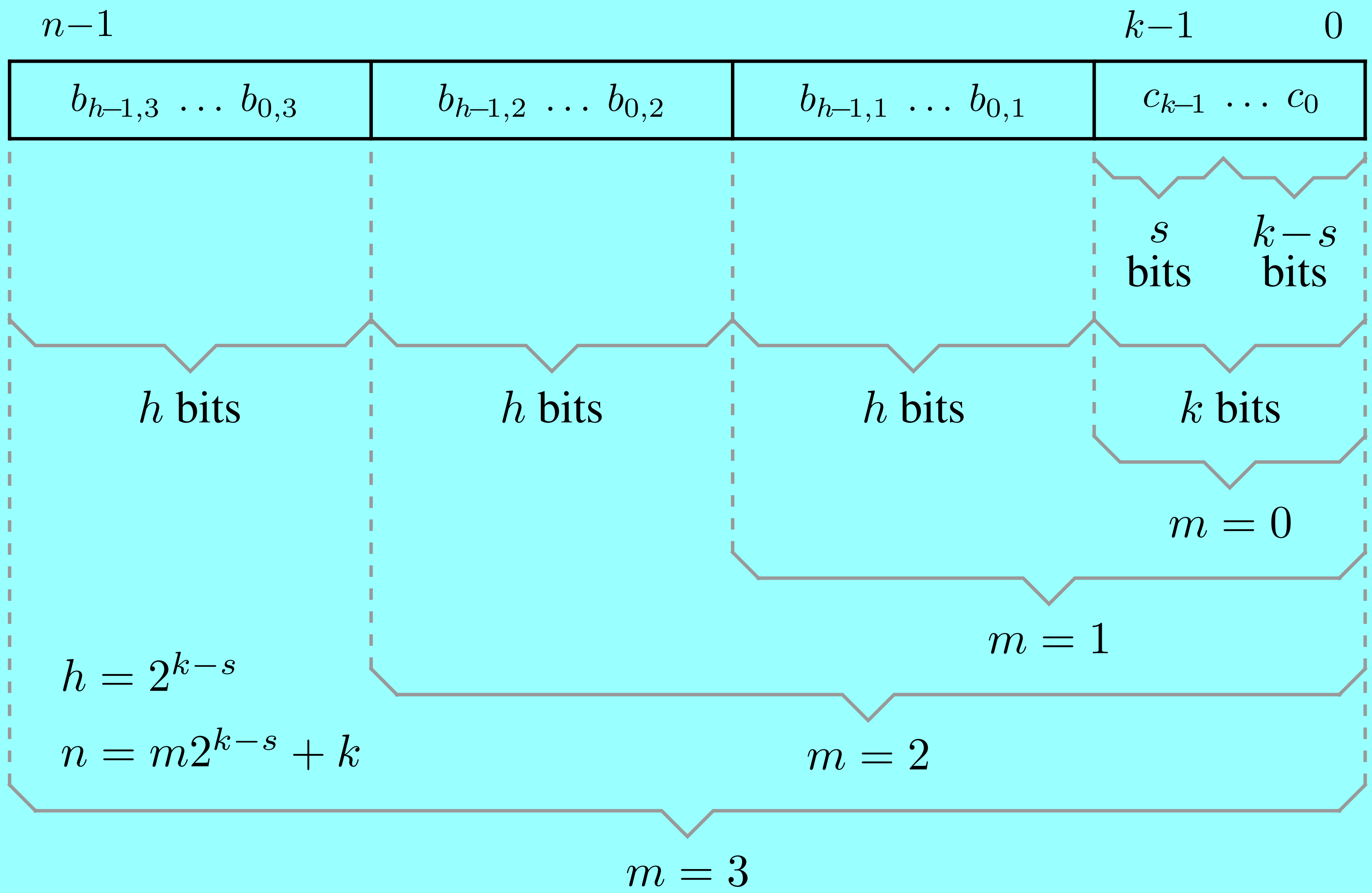
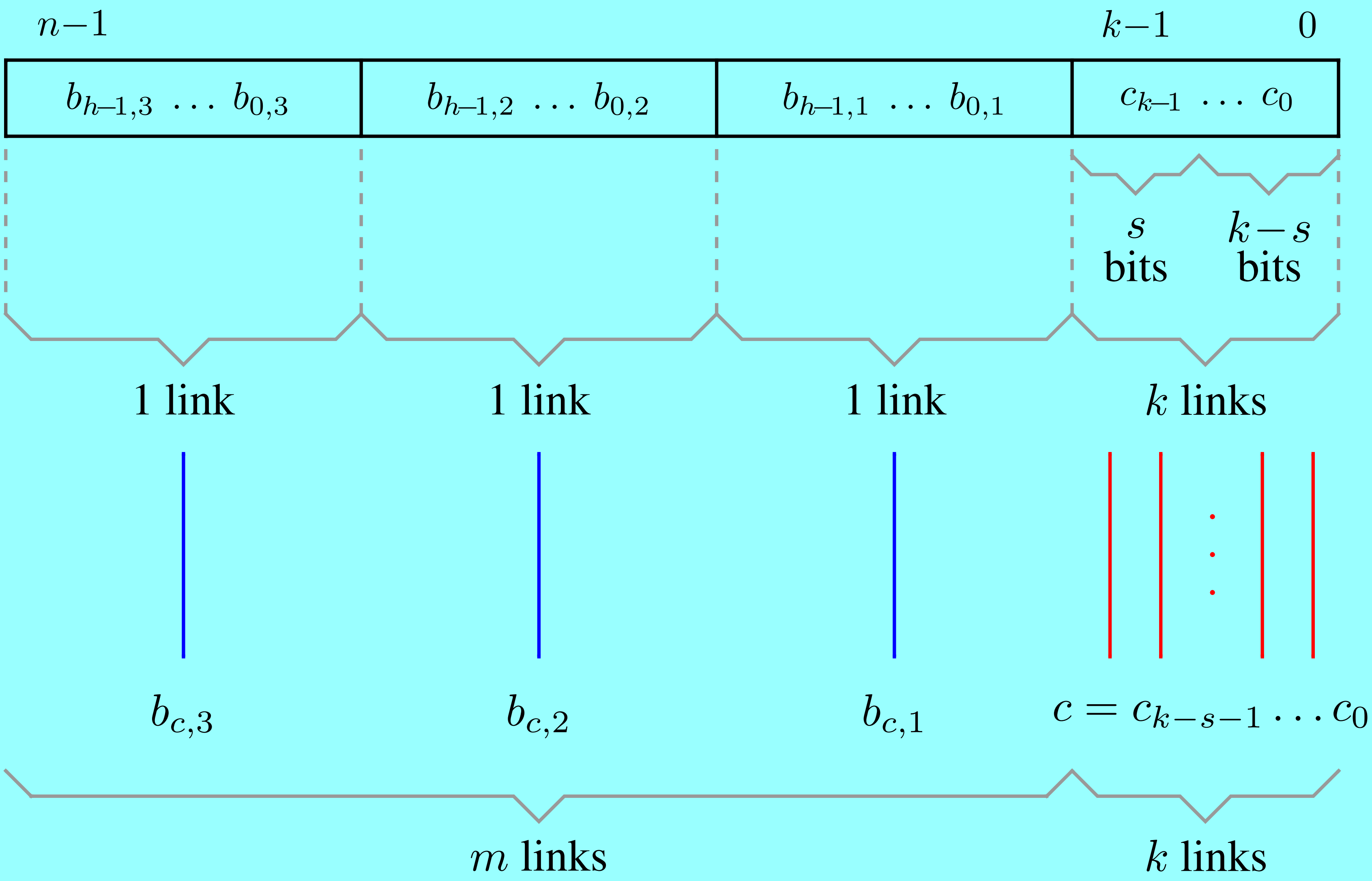
Motivations

- The supercomputer scales up year by year
- The cost of interconnection network becomes high
- Drawback of torus: long diameter
- Drawback of hypercube: large node degree
- Drawback of metacube: few configurations
- We propose a KMS-cube:
  - Low node degree $\Longrightarrow$ low hardware cost
  - Short diameter $\Longrightarrow$ fast communication
  - Ease of routing
  - Many configurations
- KMS-cube has lower cost ratio than all others

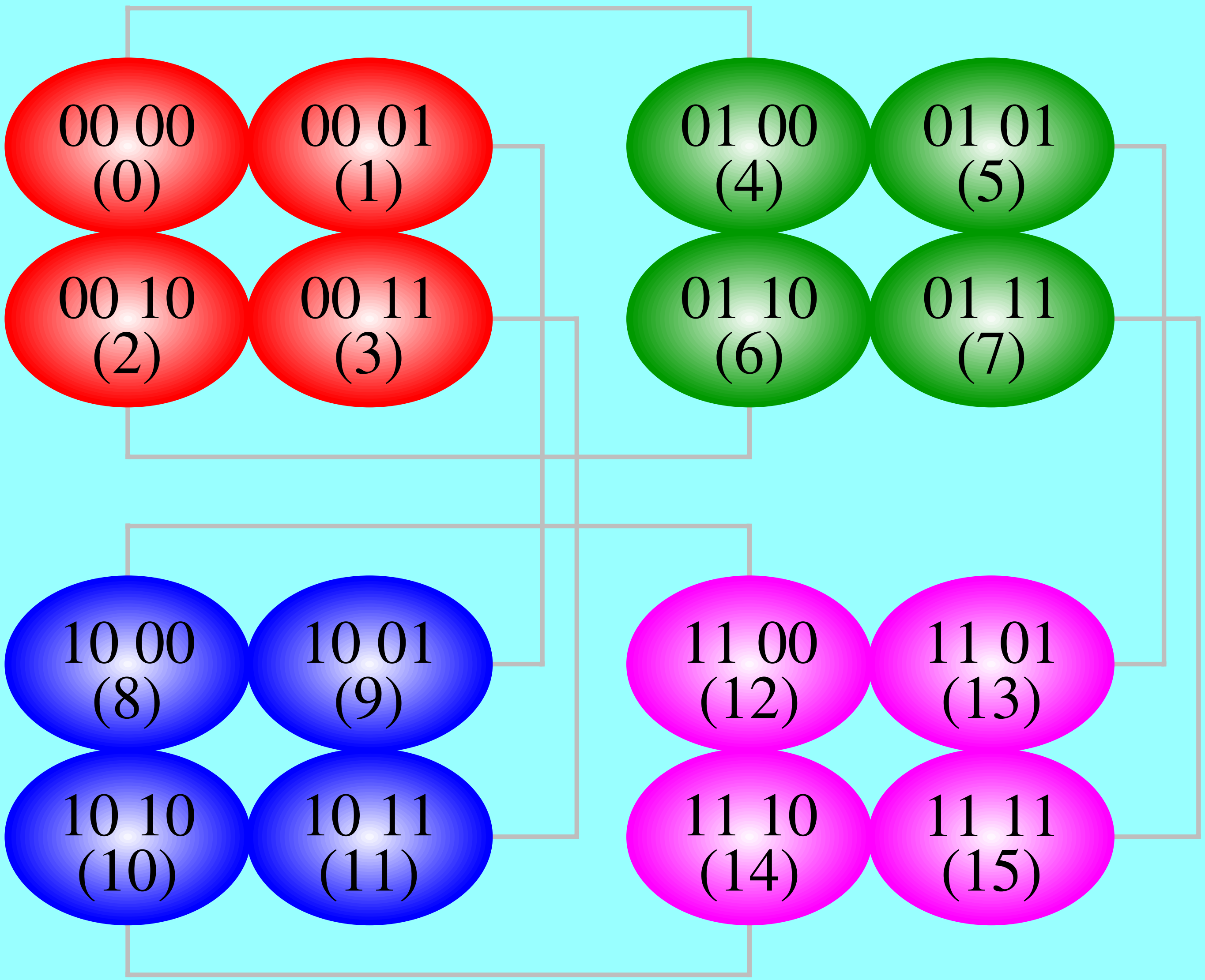
Node address of KMS-Cube



Links of KMS-Cube

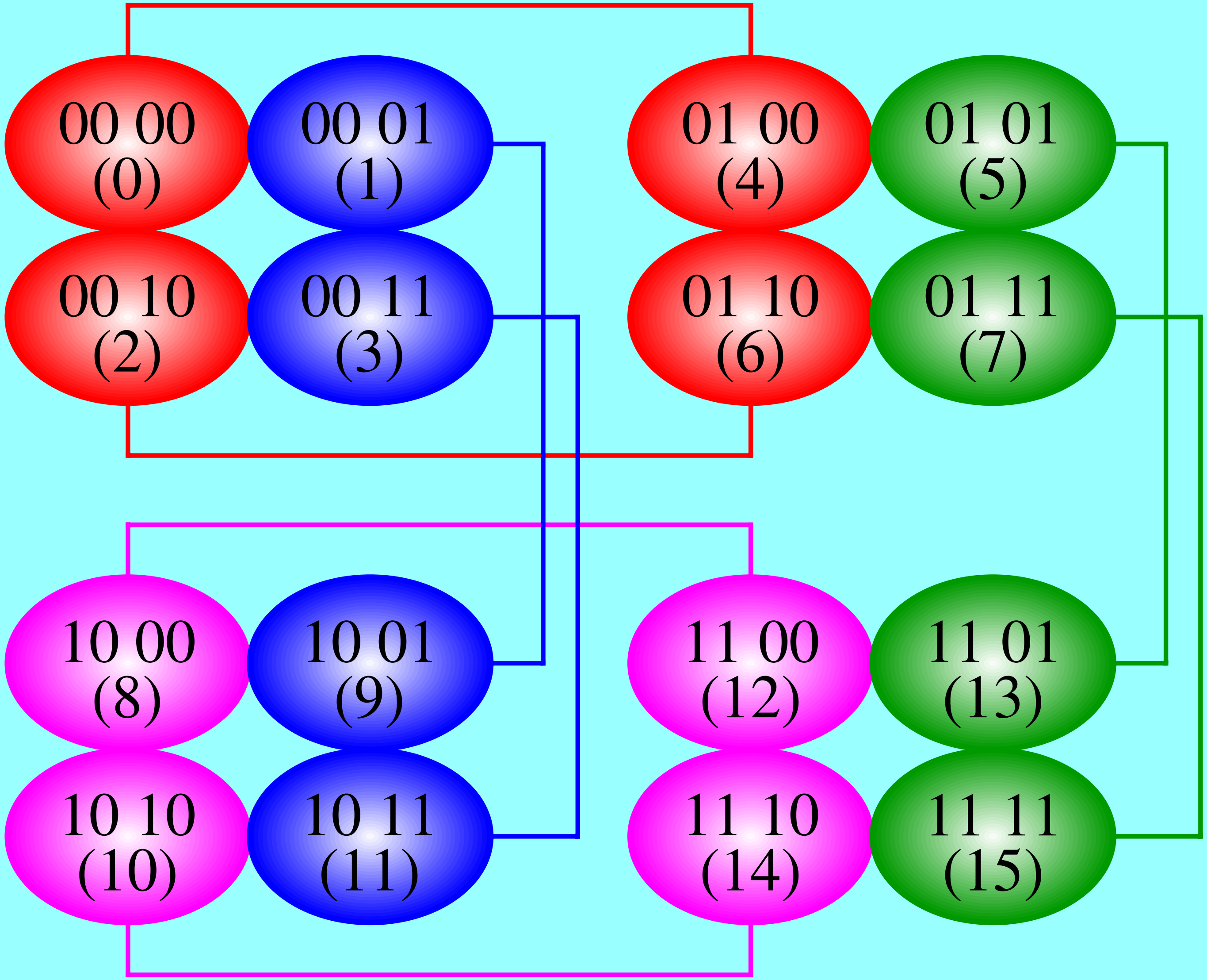


KMS-Cube with K = 2, M = 1, and S = 1



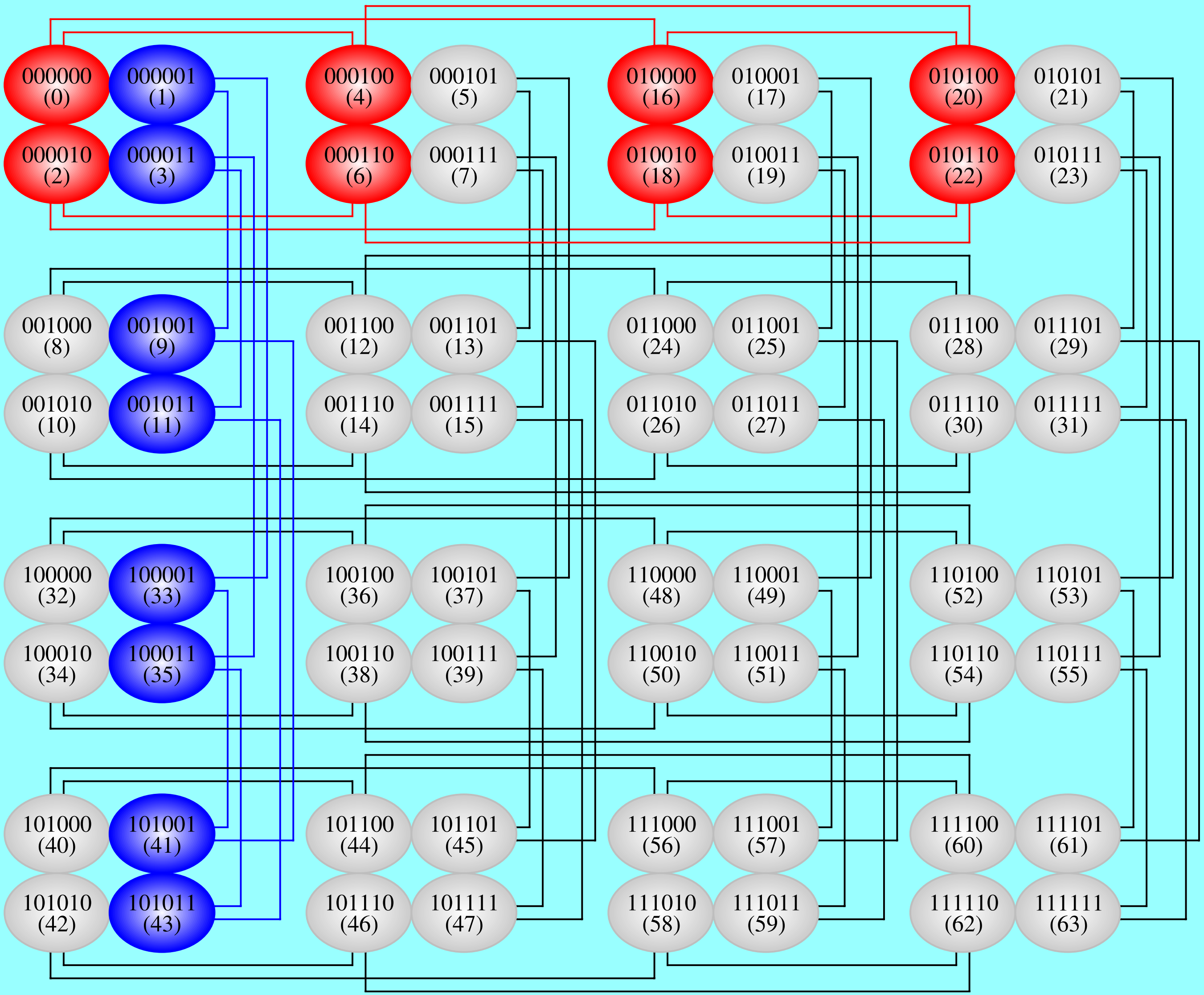
k-cube
in
same
color

KMS-Cube with K = 2, M = 1, and S = 1



(m+s)-cube
in
same
color

KMS-Cube with K = 2, M = 2, and S = 1



(m+s)-cube in same color

(coloring only 2)

KMS-Cube Routing (K = 5, M = 3, S = 2)

```
u= (00000000) (00000000) (00000000) (00  000)  →
   (00000000) (00000000) (00000001) (00  000)  →
   (00000000) (00000001) (00000001) (00  000)  →
   (00000001) (00000001) (00000001) (00  000)  →

   (00000001) (00000001) (00000001) (00  010)  →
   (00000001) (00000001) (00000101) (00  010)  →
   (00000001) (00000101) (00000101) (00  010)  →
   (00000101) (00000101) (00000101) (00  010)  →

   (00000101) (00000101) (00000101) (00  011)  →
   (00000101) (00000101) (00001101) (00  011)  →
   (00000101) (00001101) (00001101) (00  011)  →
   (00001101) (00001101) (00001101) (00  011)  →
```

KMS-Cube Routing (K = 5, M = 3, S = 2)

```
(00001101) (00001101) (00001101) (00 001)  →
(00001101) (00001101) (00001111) (00 001)  →
(00001101) (00001111) (00001111) (00 001)  →
(00001111) (00001111) (00001111) (00 001)  →

(00001111) (00001111) (00001111) (00 101)  →
(00001111) (00001111) (00101111) (00 101)  →
(00001111) (00101111) (00101111) (00 101)  →
(00101111) (00101111) (00101111) (00 101)  →

(00101111) (00101111) (00101111) (00 100)  →
(00101111) (00101111) (00111111) (00 100)  →
(00101111) (00111111) (00111111) (00 100)  →
(00111111) (00111111) (00111111) (00 100)  →
```

KMS-Cube Routing (K = 5, M = 3, S = 2)

```
(00111111)(00111111)(00111111)(00 110) →
(00111111)(00111111)(01111111)(00 110) →
(00111111)(01111111)(01111111)(00 110) →
(01111111)(01111111)(01111111)(00 110) →

(01111111)(01111111)(01111111)(00 111) →
(01111111)(01111111)(11111111)(00 111) →
(01111111)(11111111)(11111111)(00 111) →
(11111111)(11111111)(11111111)(00 111) →

(11111111)(11111111)(11111111)(00 110) →
(11111111)(11111111)(11111111)(01 110) →
(11111111)(11111111)(11111111)(11 110) = v
```

Topological Properties

Network	# Nodes	Degree	Diameter	Bisection
k-ary n-cube	k^n	$2n$	$nk/2$	$2k^{n-1}$
n-cube	2^n	n	n	2^{n-1}
MC(k, m)	2^{m2^k+k}	$m+k$	$m2^k+2^k$	2^{m2^k-1}
KMSC(k,m,s)	$2^{m2^{k-s}+k}$	$m+k$	$m2^{k-s}+2^{k-s}+s$	$2^{m2^{k-s}+s-1}$

Note: If $s = k$ or $m = 0$, the KMSC(k,m,s) becomes an n-cube

Cost Ratio

$$CR(G) = \frac{d(G) \times w_1 + D(G) \times w_2}{\log_2 |(G)|}$$

$d(G)$ is the node degree of G
$D(G)$ is the diameter of G
$|(G)|$ is the total number of nodes in G
w_1 and w_2 are weights with $w_1 + w_2 = 100\%$

The CR of an n-cube is

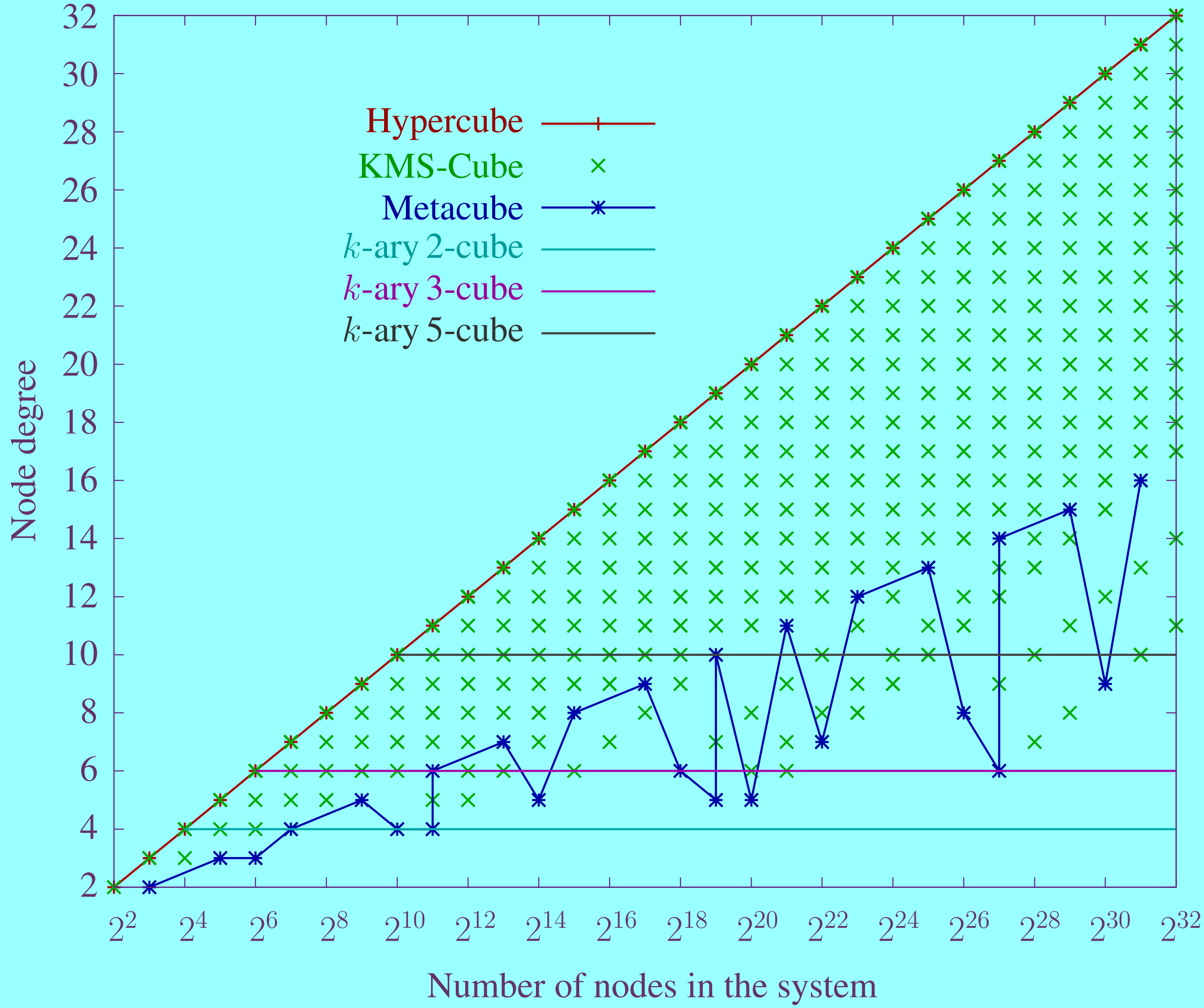
$$CR(n\text{-cube}) = \frac{n \times w_1 + n \times w_2}{n} = w_1 + w_2 = 1$$

irrespective of n, w_1, and w_2

Network Examples

Network	N	d	D	CR
32-ary 3-cube	32,768	6	48	1.800
15-cube	32,768	15	15	1.000
MC(1,7)	32,768	8	16	0.800
KMSC(3,7,1)	32,768	6	17	0.767
128-ary 3-cube	2,097,152	6	192	4.714
23-cube	8,388,608	23	23	1.000
MC(1,11)	8,388,608	12	24	0.783
KMSC(3,5,1)	8,388,608	8	25	0.717
1024-ary 3-cube	1,073,741,824	6	1,536	25.700
31-cube	2,147,483,648	31	31	1.000
MC(1,15)	2,147,483,648	16	32	0.774
KMSC(3,7,1)	2,147,483,648	10	33	0.694

Degree Comparison

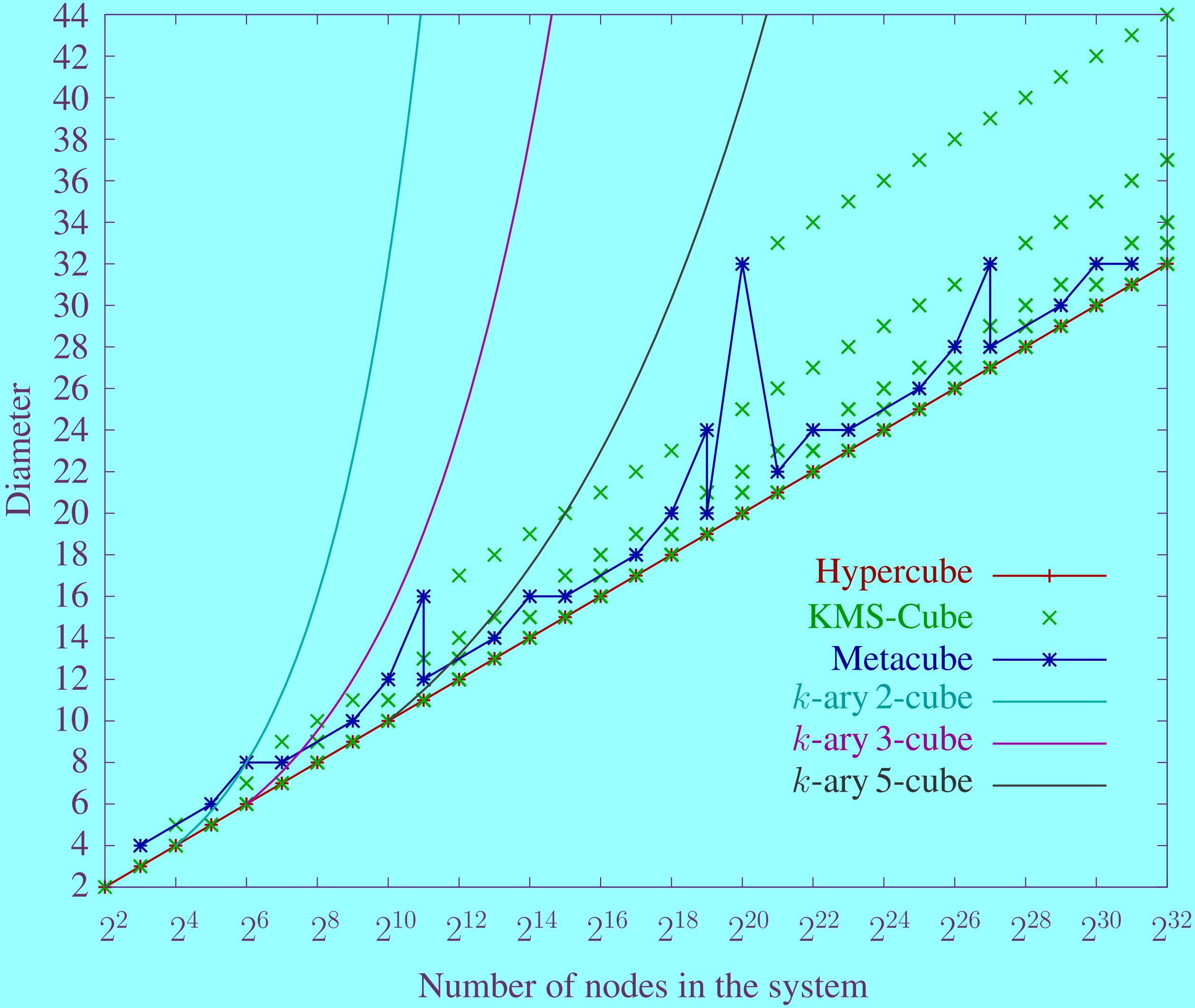


KMS-Cube:

Lower degree than hypercubes

Has much more configurations than metacubes

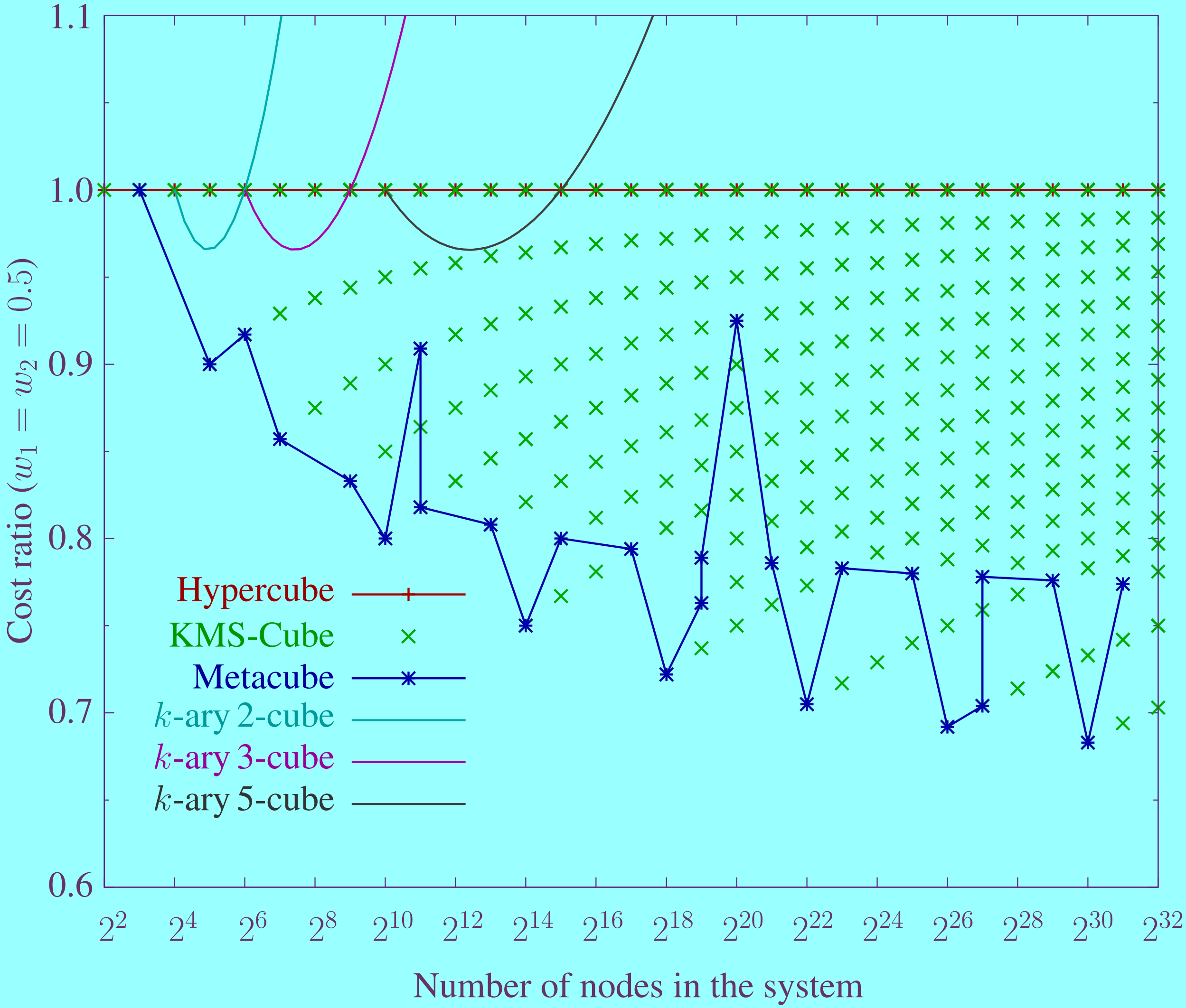
Diameter Comparison



KMS-Cube:

Shorter diameter than torus

Cost Ratio Comparison



KMS-Cube:

Lower cost than others

Has much more configurations than metacubes

Efficient routing

Conclusions

- Node degree and diameter have a great impact on the cost and performance of networks
- We proposed a KMS-cube which has following features
  - Low node degree $\Longrightarrow$ low harware cost
  - Short diameter $\Longrightarrow$ fast communication
  - Ease of routing
  - Many configurations
  - Lower cost ratio than all others
- Future works
  - Develop a routing algorithm to find a shortest path
  - Check whether it is Hamiltonian