Adjusting Parameters of k-Ary n-Cube to Achieve Better Cost Performance

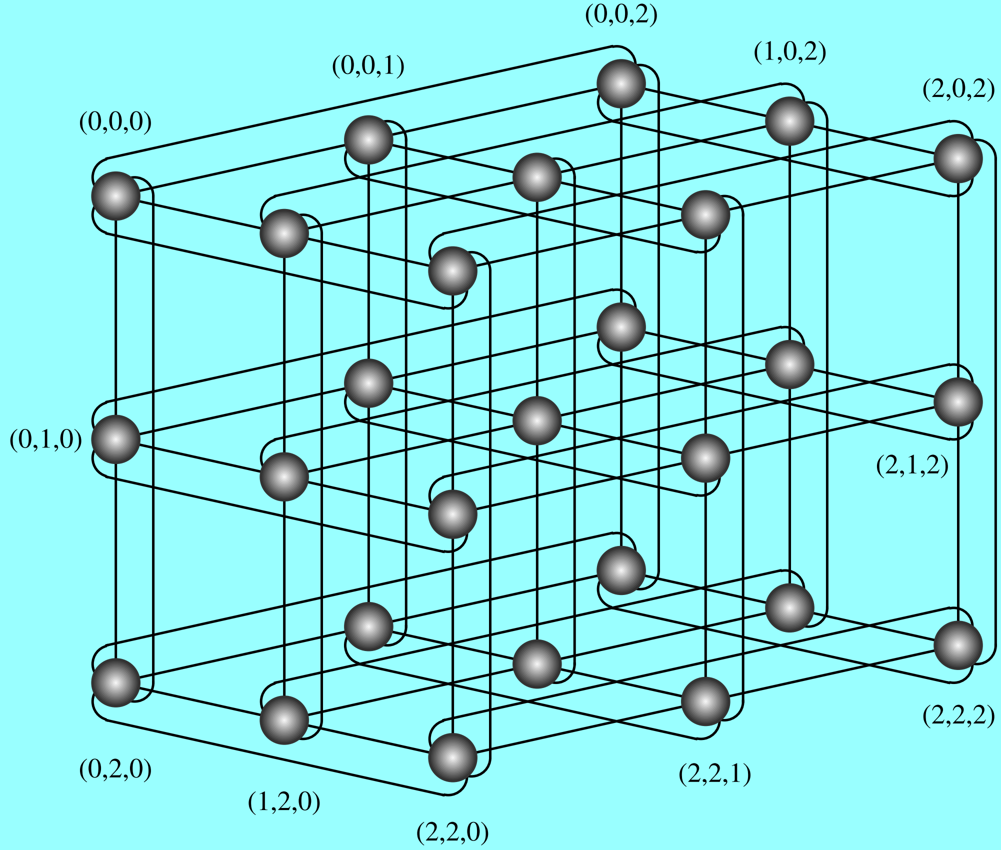
Yamin Li

Dept. of Computer Science
Hosei University

http://cis.k.hosei.ac.jp/∼yamin/

Question about k-Ary n-Cube

- If we have 27 nodes, we can build a 3-ary 3-cube

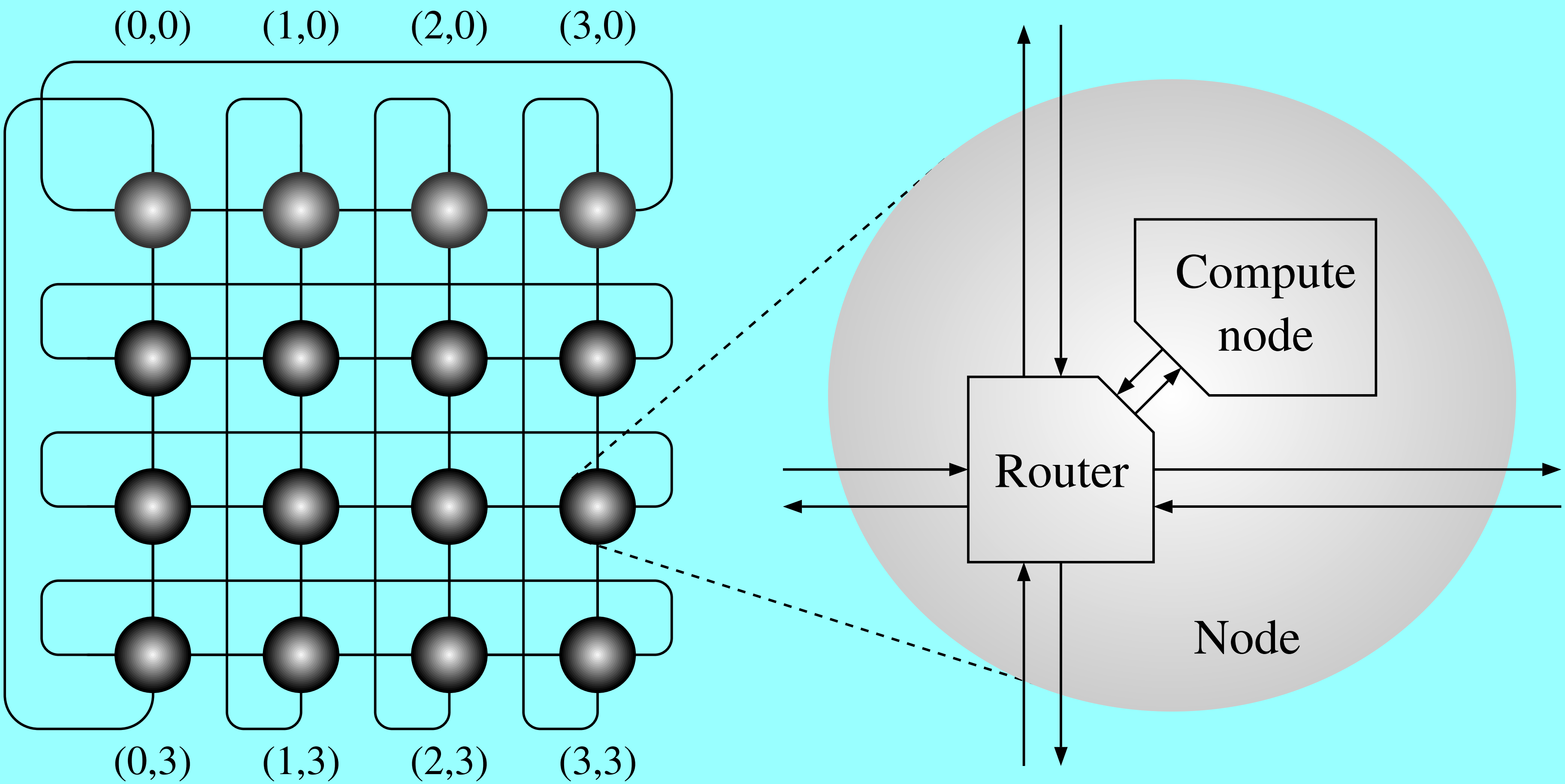


$k = 3, n = 3$

$N = k^n = 27$

- Suppose we have 10,000,000 nodes, $k = ?$, $n = ?$ so that the system has high performance at low cost

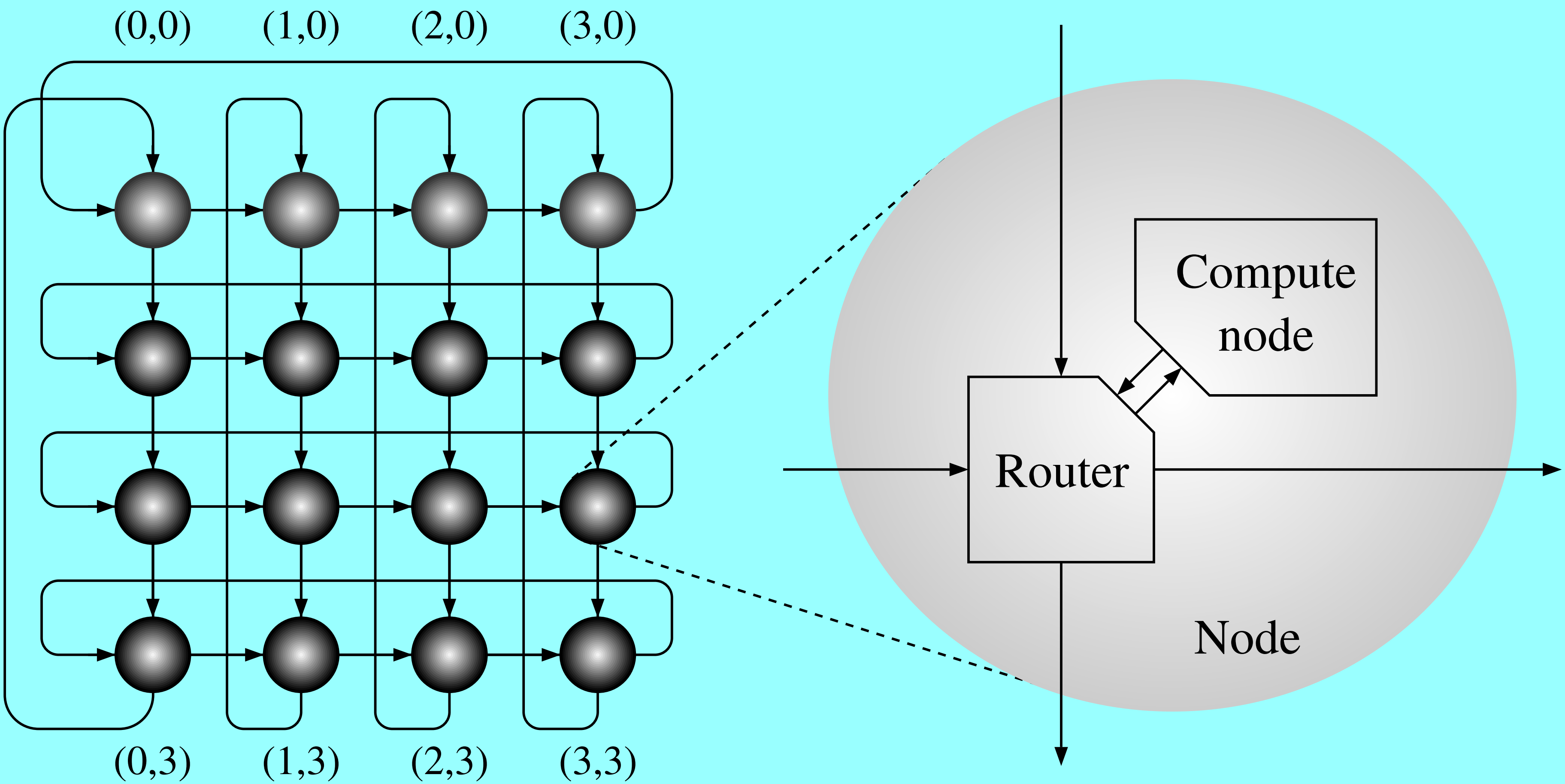
Bidirectional 4-Ary 2-Cube ($p = 1$)



External ports: 4

Internal ports: 1 p: the number of computer nodes in a node

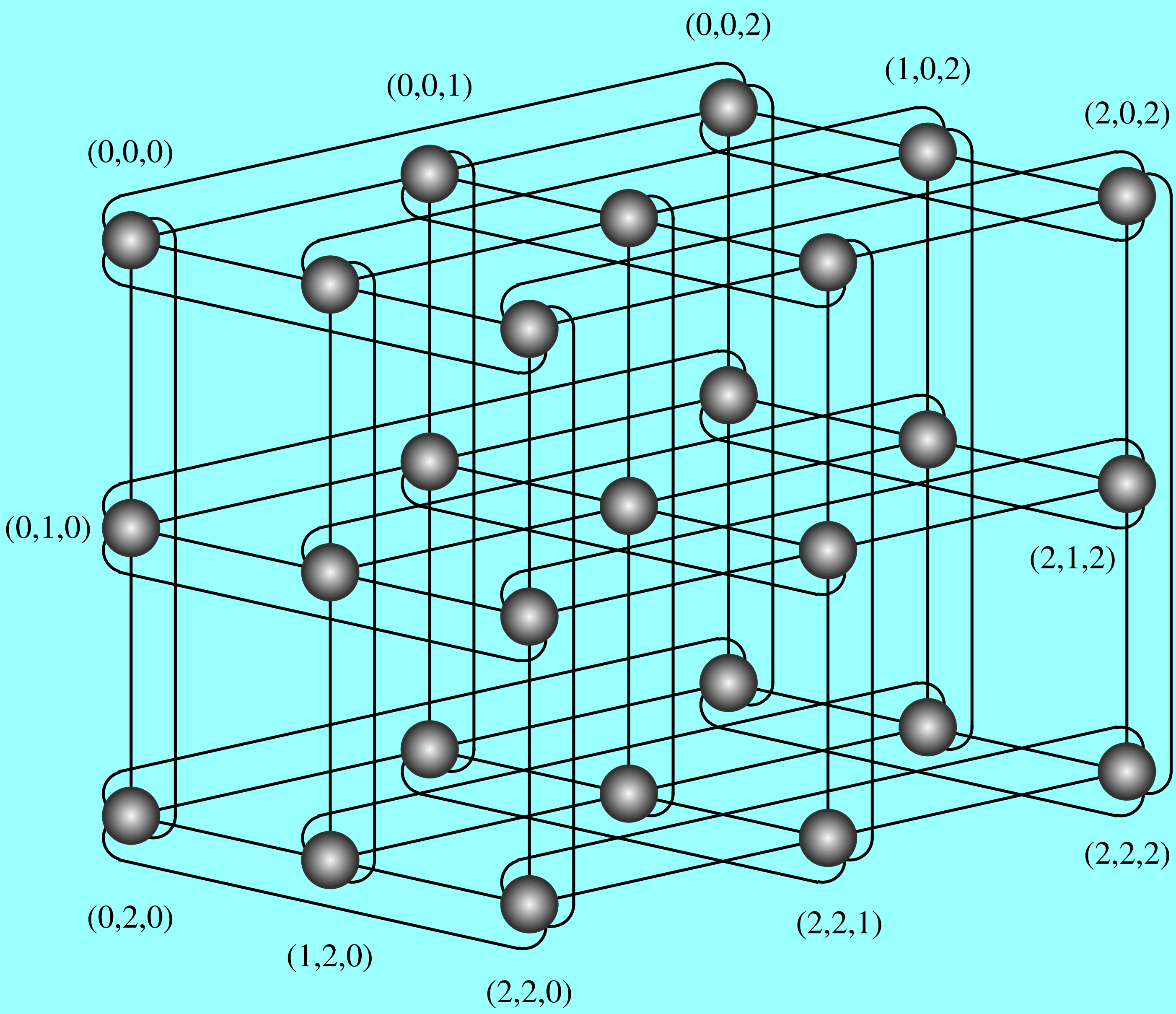
Unidirectional 4-Ary 2-Cube ($p = 1$)



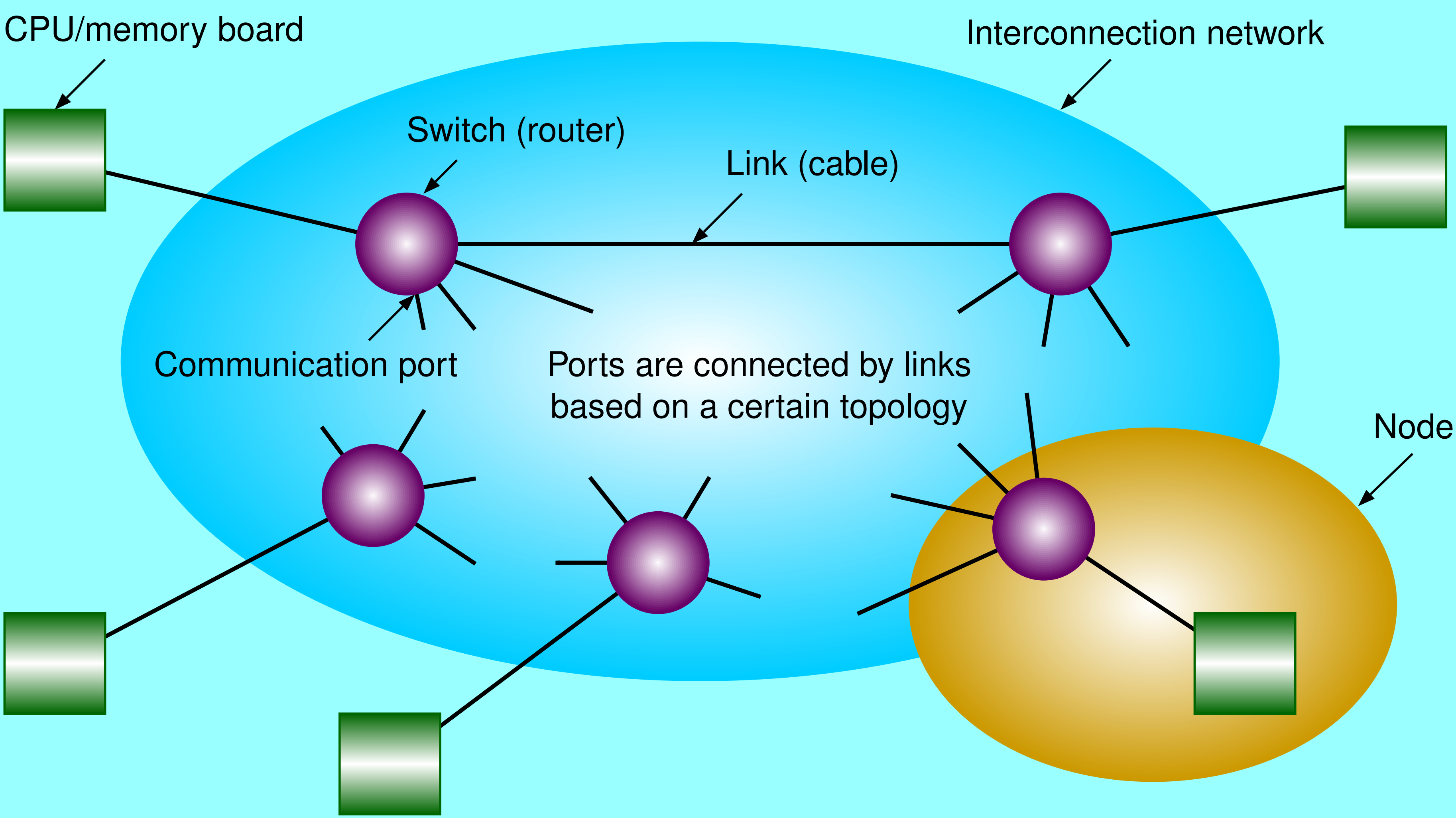
External ports: 2

Internal ports: 1 p: the number of computer nodes in a node

Bidirectional or Unidirectional 3-Ary 3-Cube

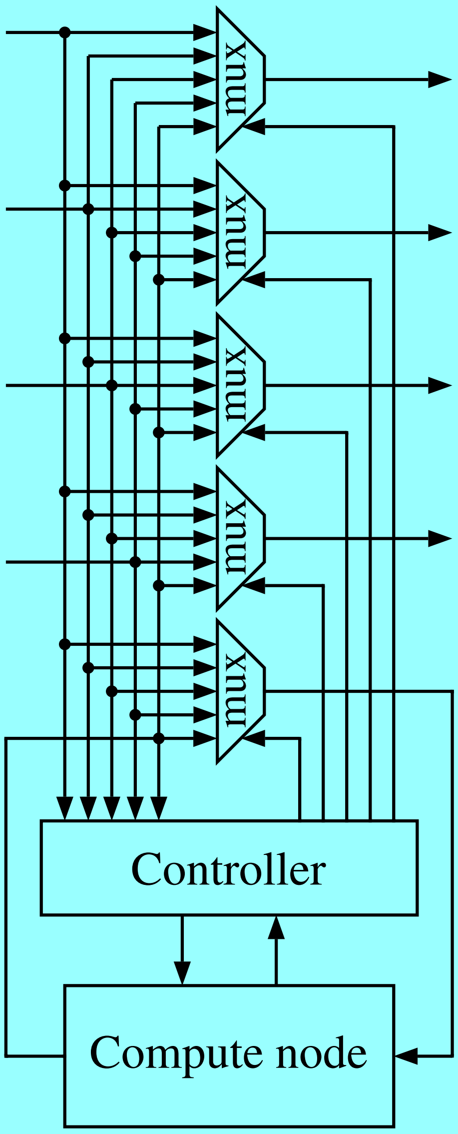


Interconnection Network

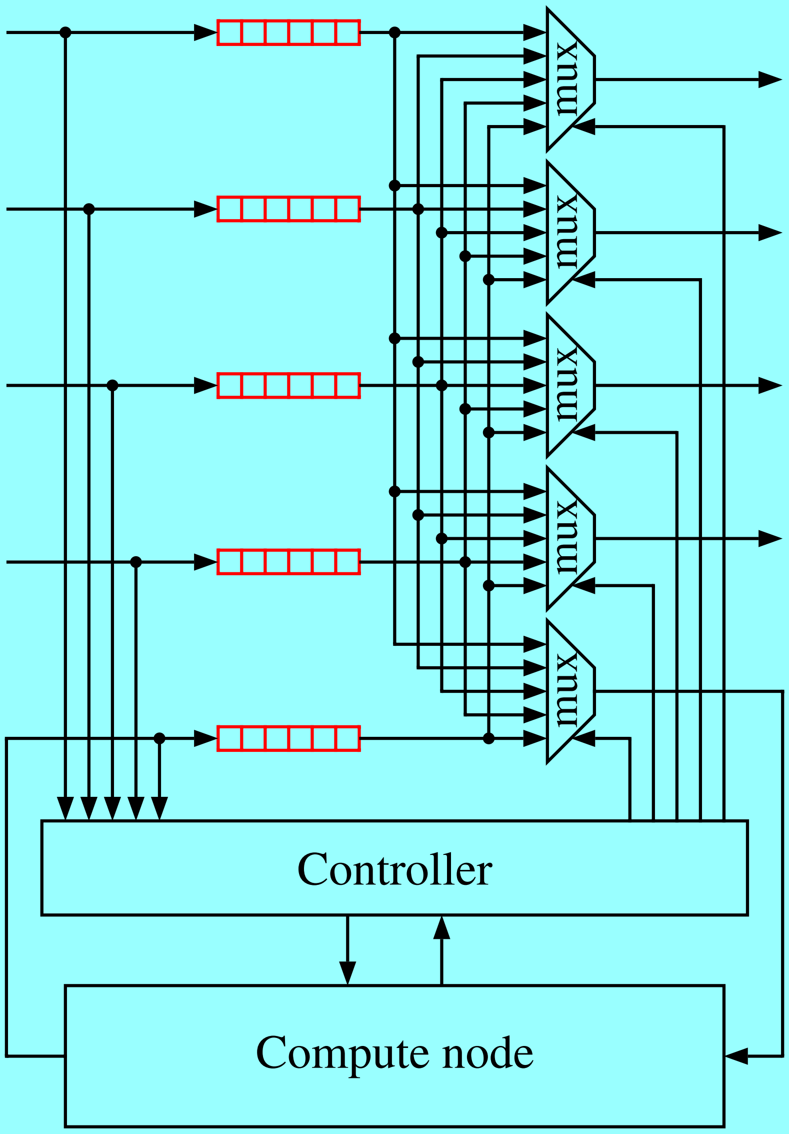


Used for designing large distributed memory parallel systems

Router with Four External Ports ($p = 1$)

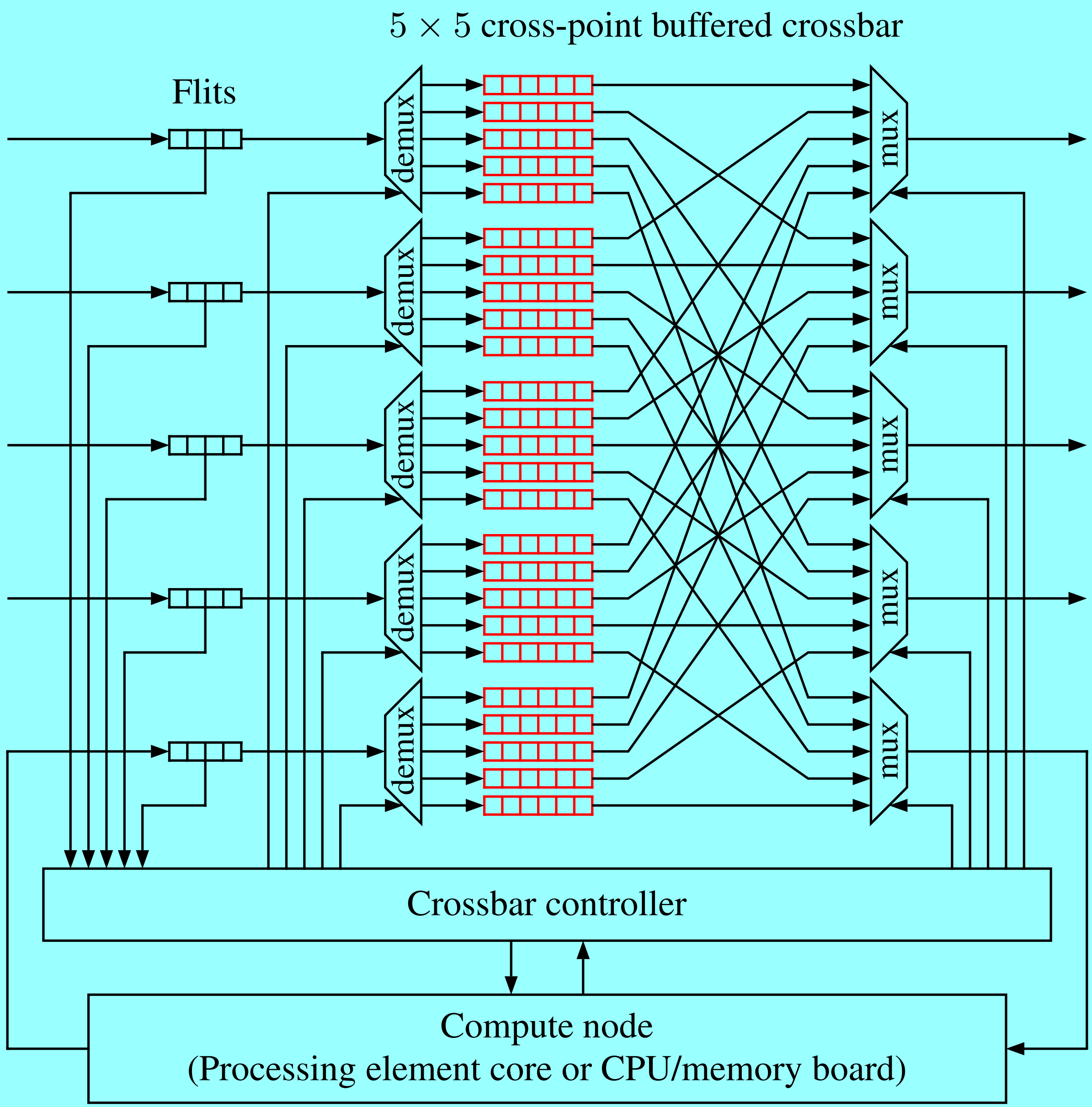


(a) 5×5 crossbar

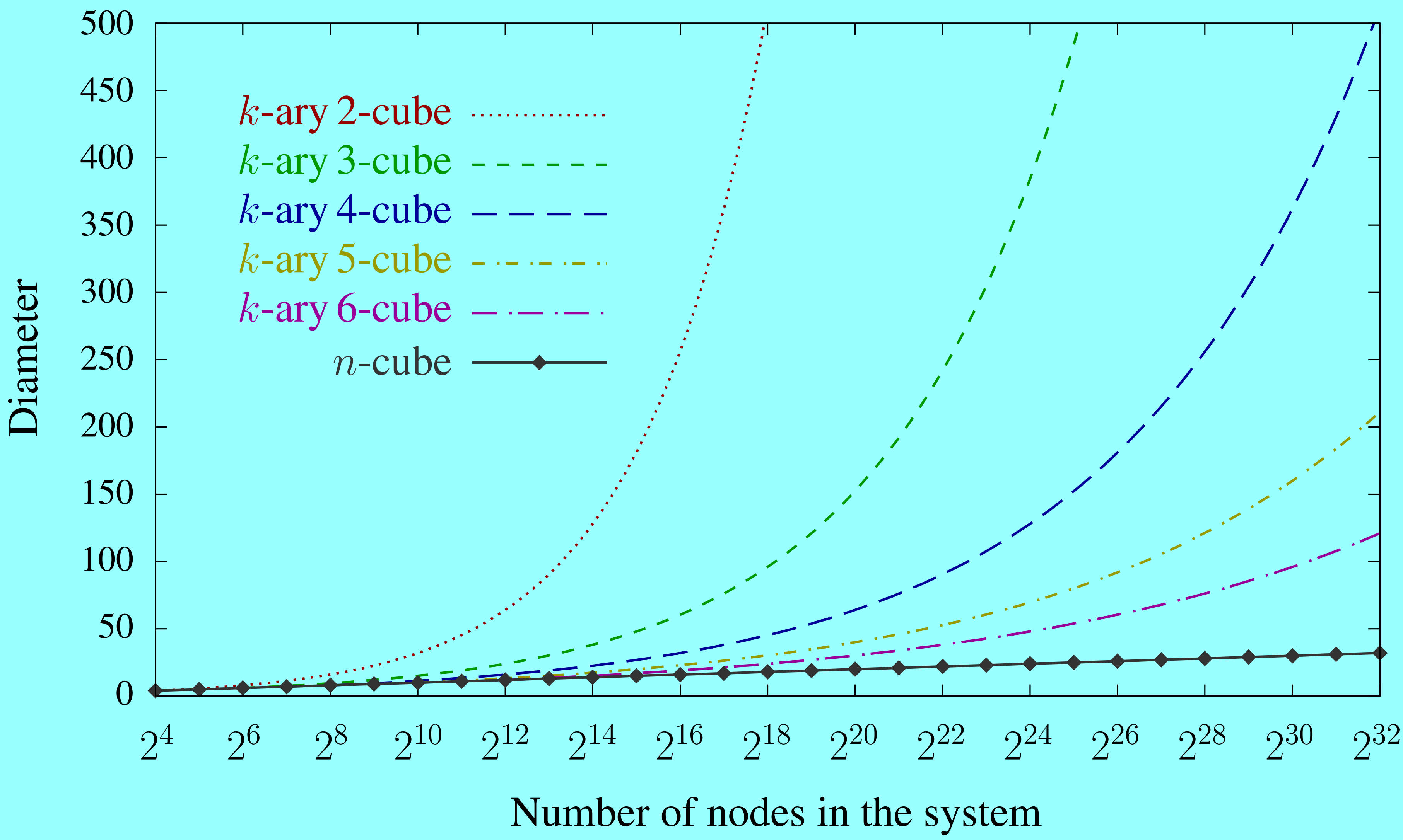


(b) 5×5 input buffered crossbar

Cross-Point Buffered Router ($p = 1$)



Diameter Comparison (Bidirectional Torus)

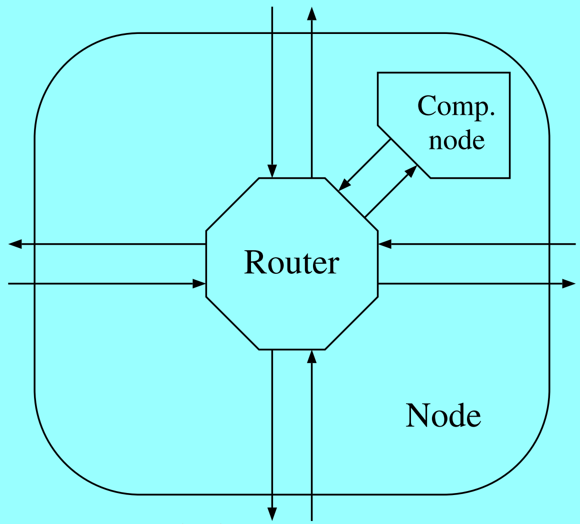


As n becomes larger, the diameter becomes smaller, but degree gets larger

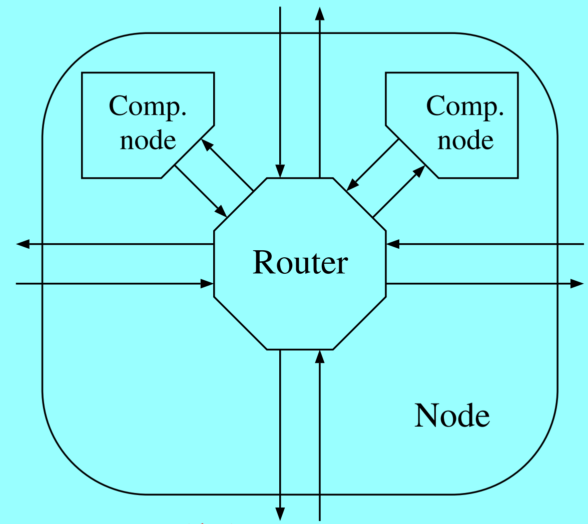
Topological Properties

Network	# of nodes	Degree	Diameter	Bisection
n-cube	2^n	n	n	2^{n-1}
k-ary n-cube (mesh)	k^n	$2n$	$n(k-1)$	k^{n-1}
Bidirectional k-ary n-cube (torus)	k^n	$2n$	$n\lfloor k/2 \rfloor$	$2k^{n-1}$
Unidirectional k-ary n-cube (torus)	k^n	n	$n(k-1)$	k^{n-1}

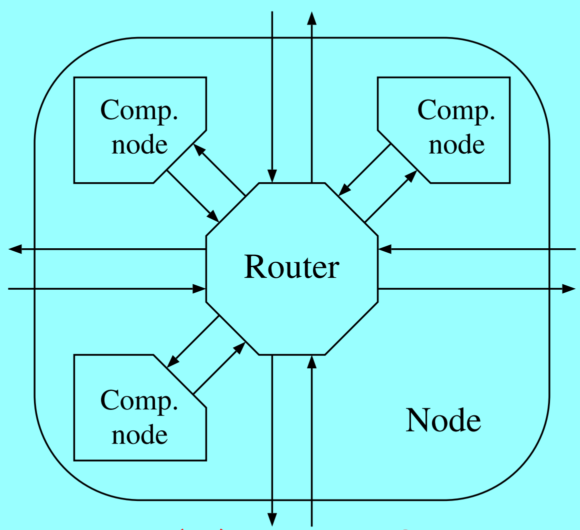
Number of Compute Nodes in a Node



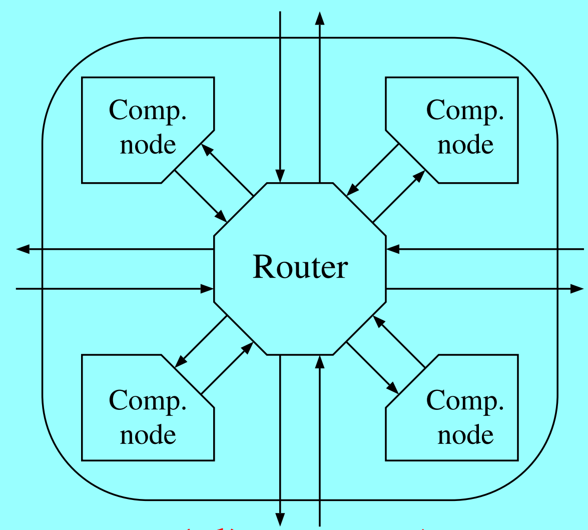
(a) $p = 1$



(b) $p = 2$



(c) $p = 3$



(d) $p = 4$

RCP — Relative Cost Performance

$$RCP = \frac{(d+p)^{\lambda} D}{(\log_2 N + p)^{\lambda} \log_2 N}$$

d: node degree
p: the number of compute nodes in a node
λ: the router complexity $(1.0 \leq \lambda \leq 2.0)$
D: diameter
N: the number of nodes in system

Taking p and λ into consideration
The smaller RCP, the lower cost and higher performance

RCP of Hypercube

$$RCP = \frac{(d+p)^{\lambda} D}{(\log_2 N + p)^{\lambda} \log_2 N}$$
$$= \frac{(n+p)^{\lambda} n}{(n+p)^{\lambda} n}$$
$$\equiv 1$$

n-cube:

$d = n$ (node degree)
$D = n$ (diameter)
$N = 2^n$ (the number of nodes in system)

Irrespective of λ, p, and N

Derivative of RCP

Let $x = \log_2 N$, then $N = 2^x = k^n$, or $k = 2^{x/n}$, therefore we have $D = kn/2 = 2^{x/n} n/2$

Let

$$g(x) = (2n+p)^{\lambda} 2^{x/n} n/2$$
$$f(x) = (x+p)^{\lambda} x$$

Then $RCP' = (g(x)/f(x))' = \dfrac{g'(x)f(x) - g(x)f'(x)}{f^2(x)}$

where

$$g'(x) = (2n+p)^{\lambda} 2^{x/n} \ln 2/2$$
$$f'(x) = ((x+p)^{\lambda})' x + (x+p)^{\lambda} x'$$
$$= \lambda (x+p)^{\lambda-1} x + (x+p)^{\lambda}$$

Derivative of RCP

Let $RCP' = 0$, i.e.,

$$g'(x)f(x) = g(x)f'(x)$$

The positive number of x can be calculated from the equation

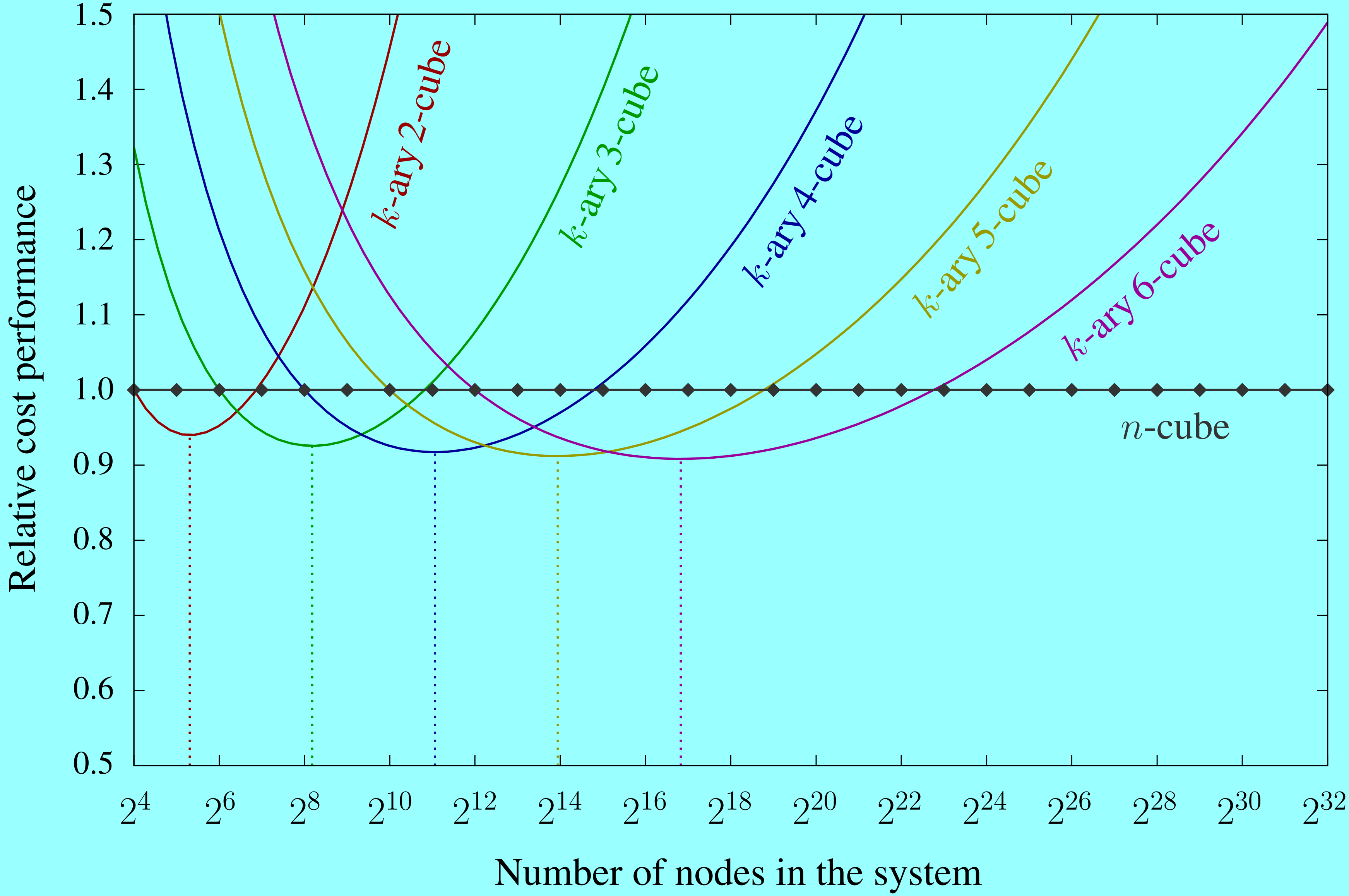
$$\ln 2(x+p)x = n((\lambda+1)x+p)$$

Then we can determine an odd k from the equation

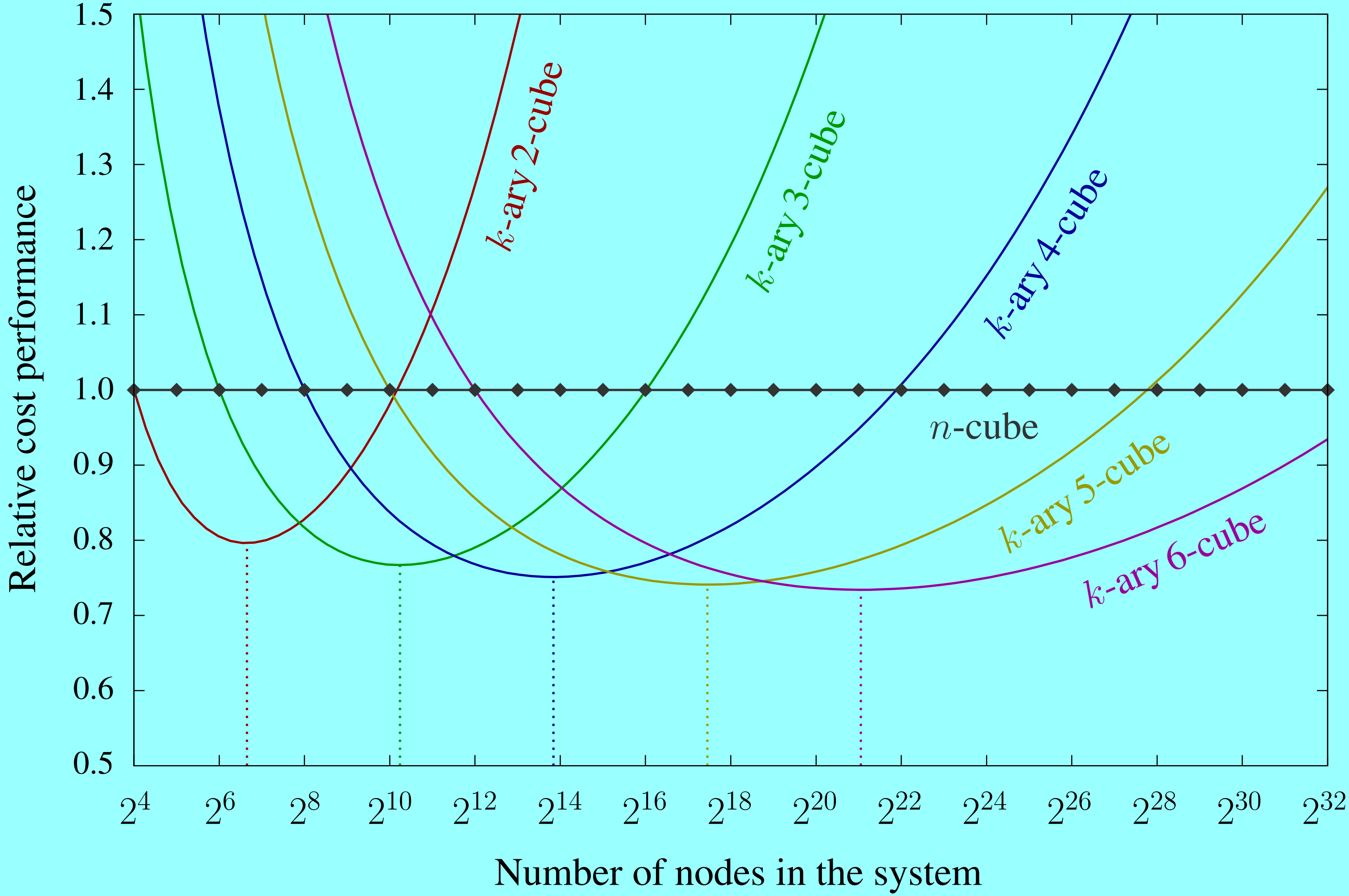
$$k = \lfloor 2^{x/n} \rfloor \text{ or}$$
$$k = \lceil 2^{x/n} \rceil$$

If both are even, $k = 2^{x/n} + 1$

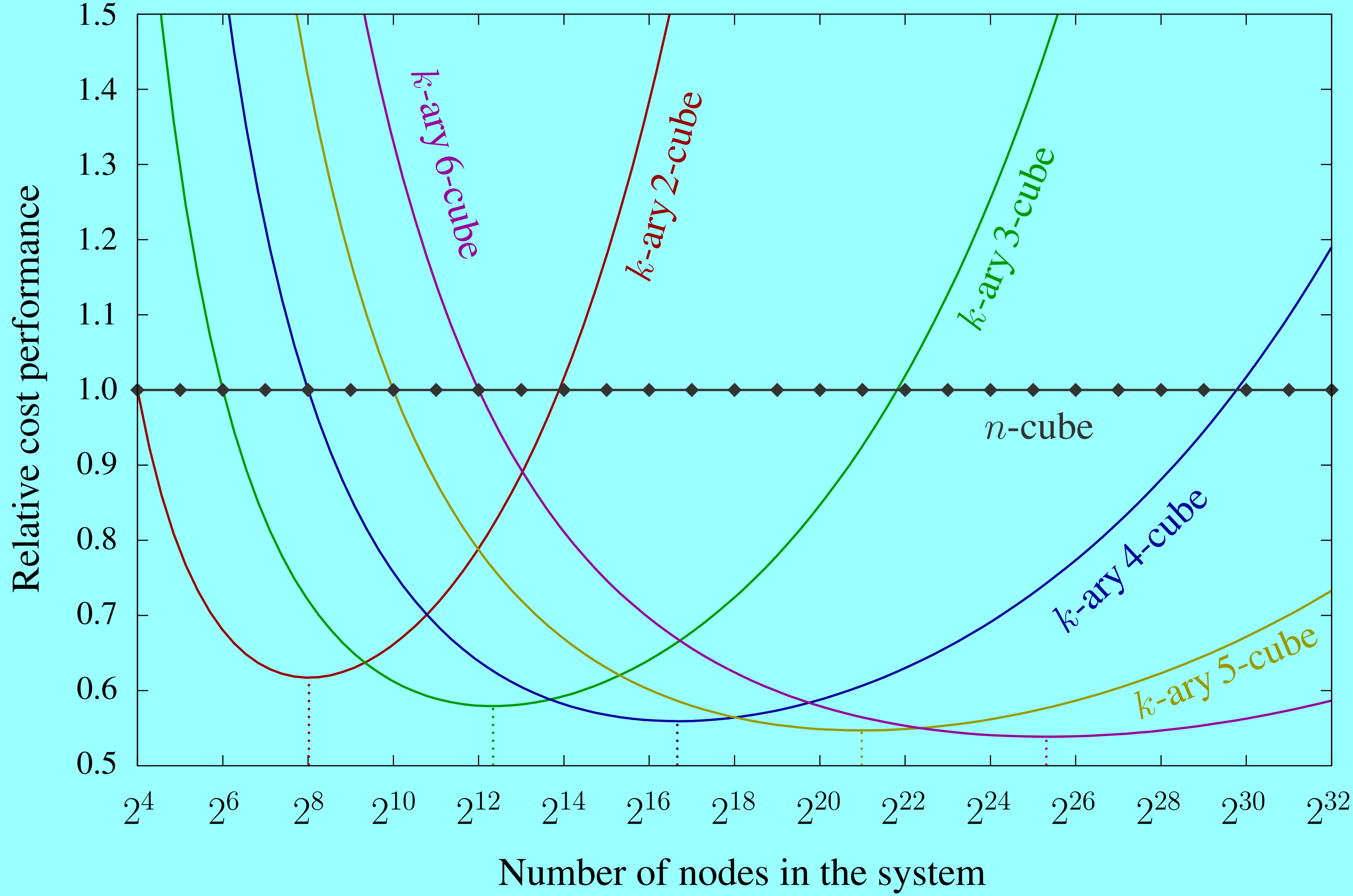
RCP Comparison ($p = 1, \lambda = 1.0$)



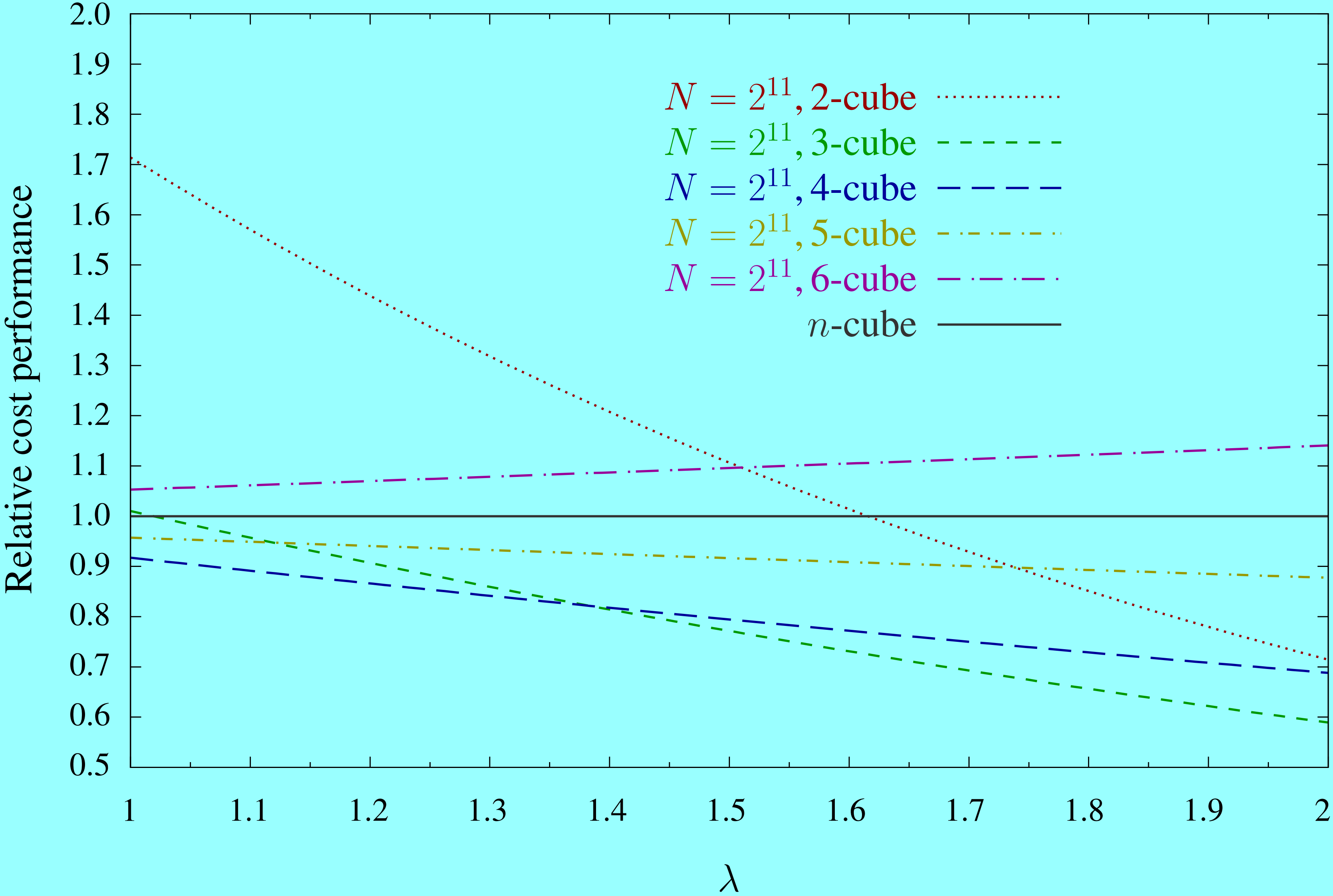
RCP Comparison ($p = 1, \lambda = 1.5$)



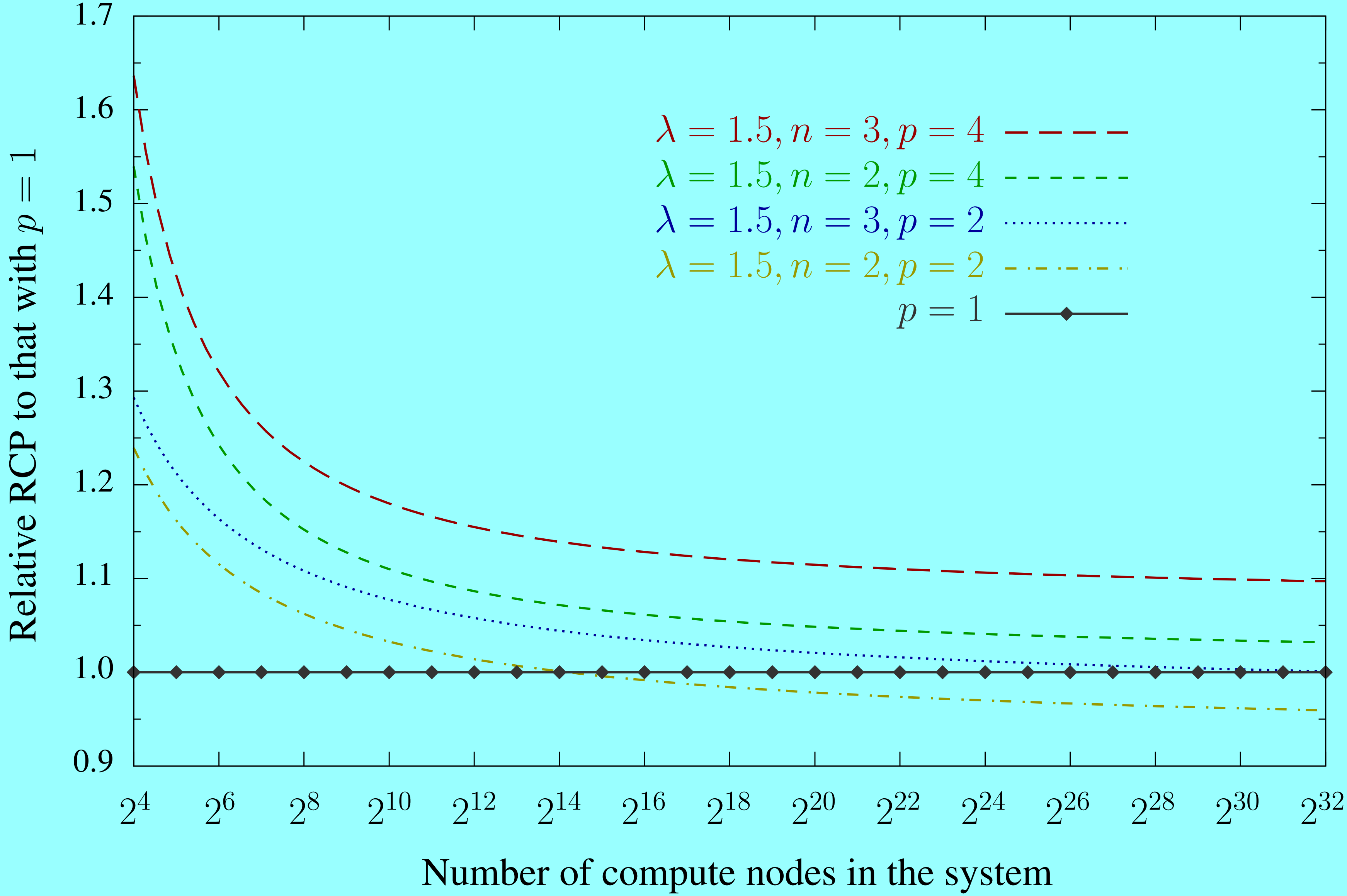
RCP Comparison ($p = 1, \lambda = 2.0$)



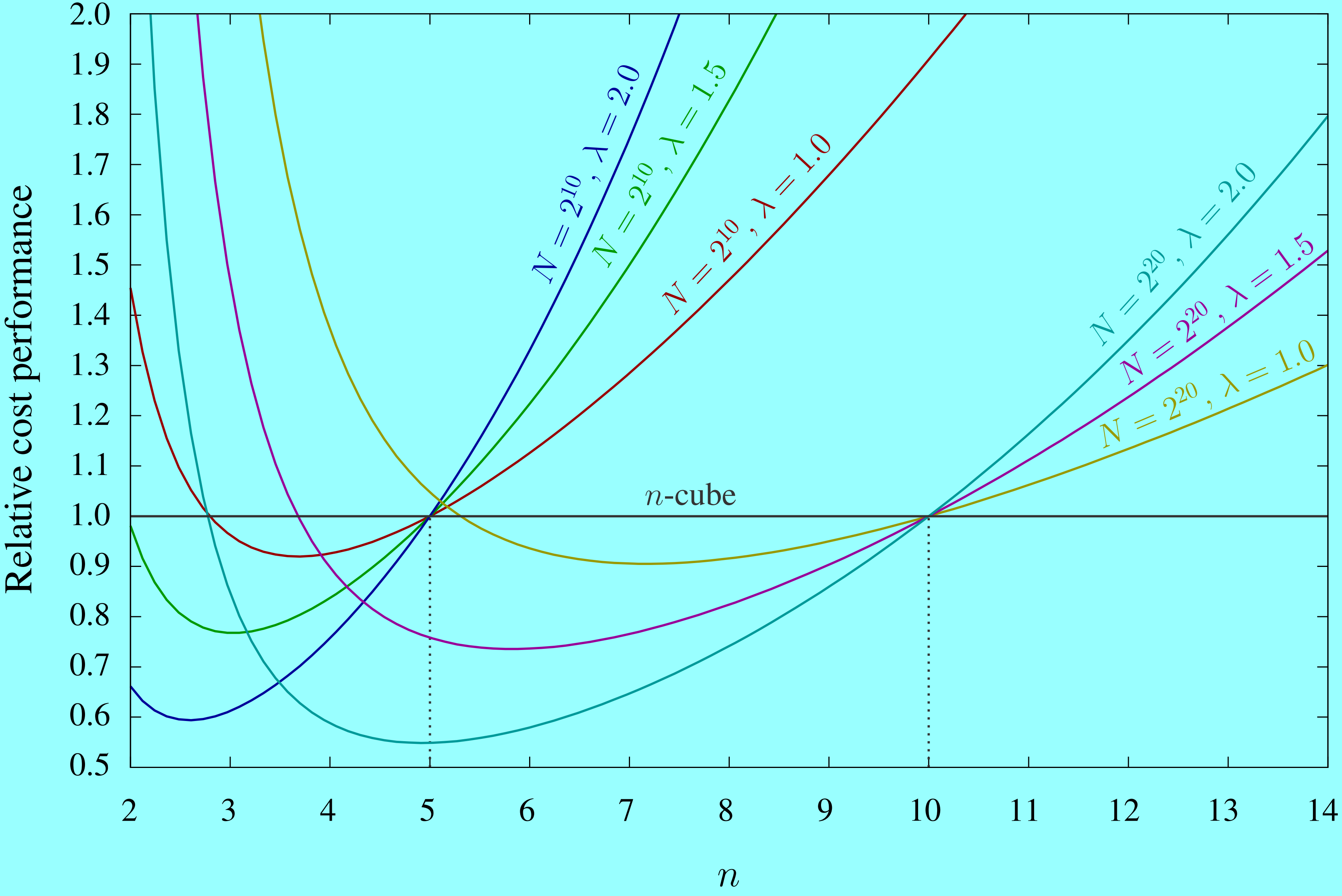
RCP Comparison on λ ($p = 1$)



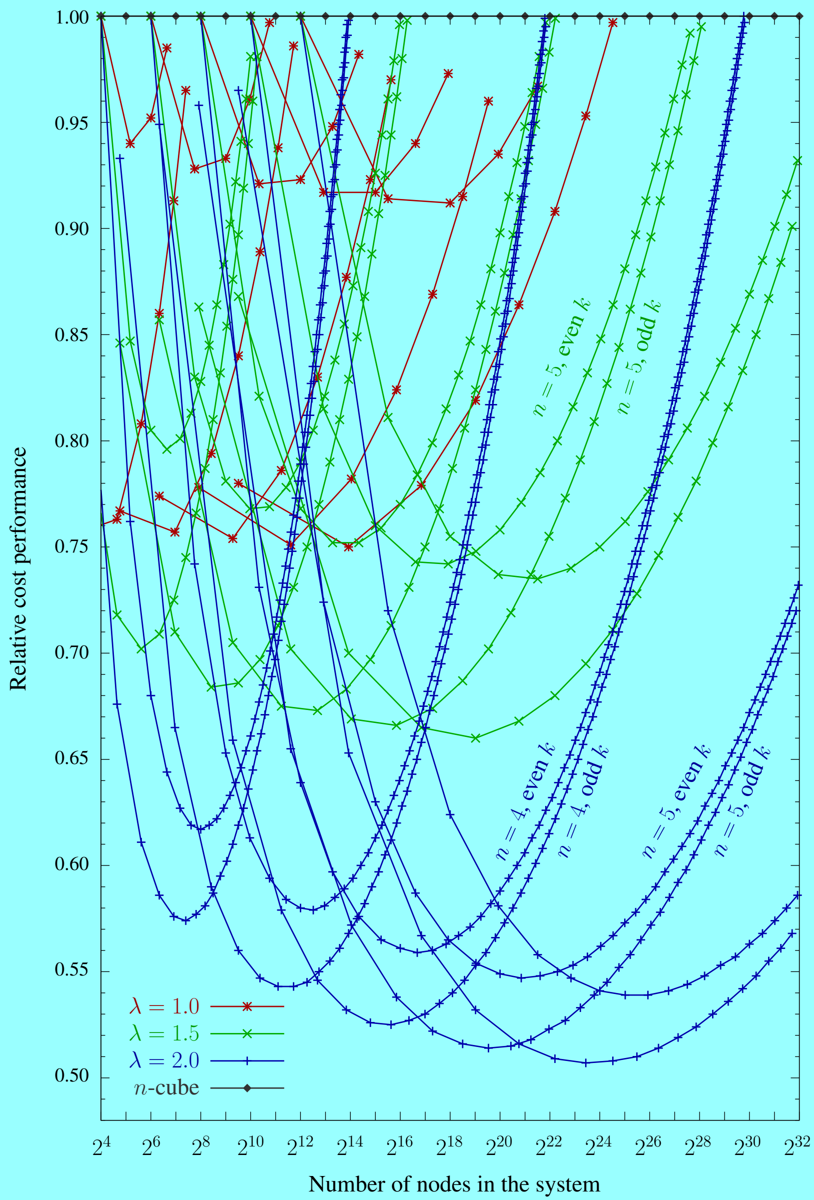
RCP Comparison on p



RCP Comparison on n ($p = 1$)



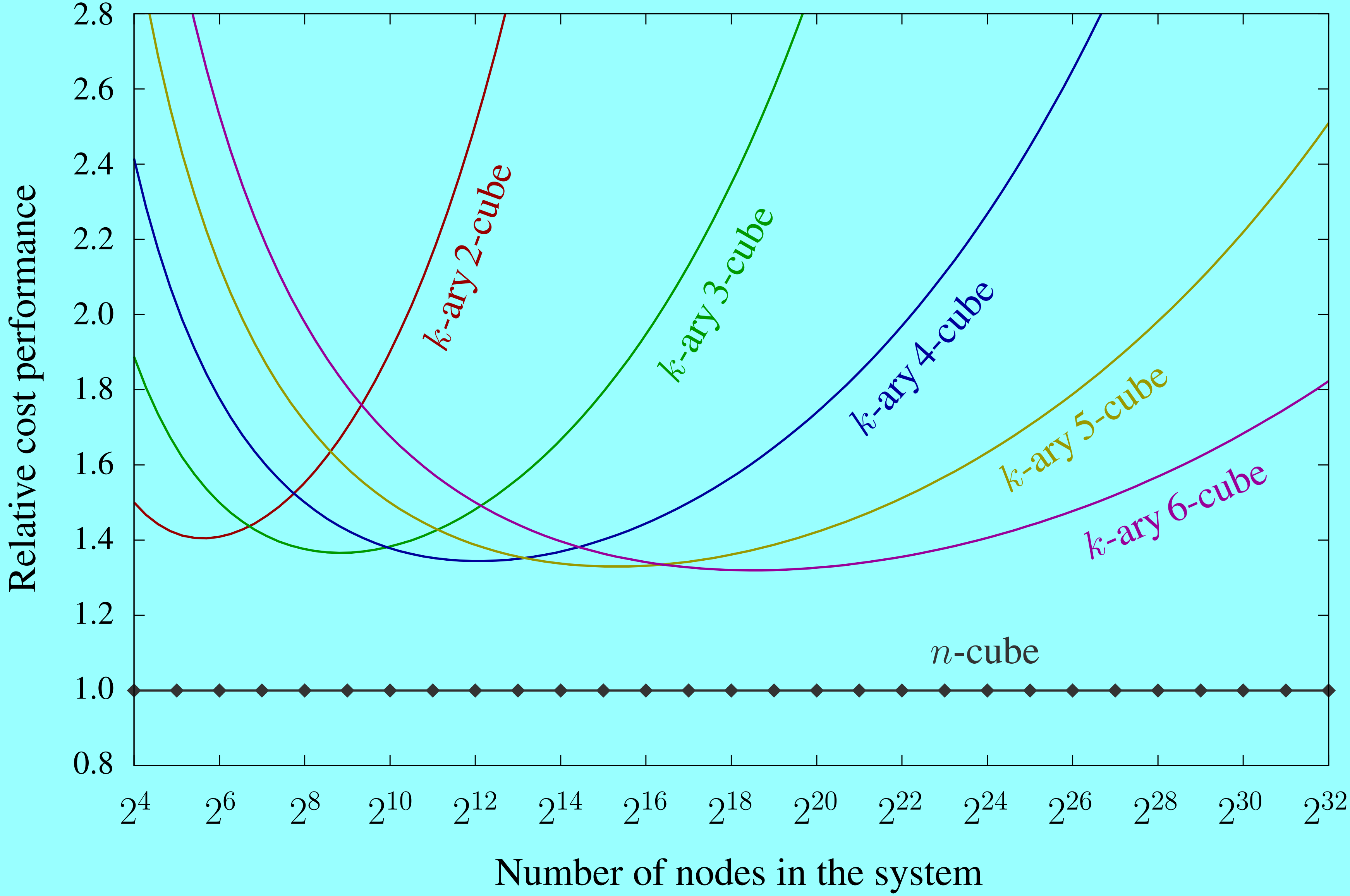
RCP Comparison for Bidirectional Torus



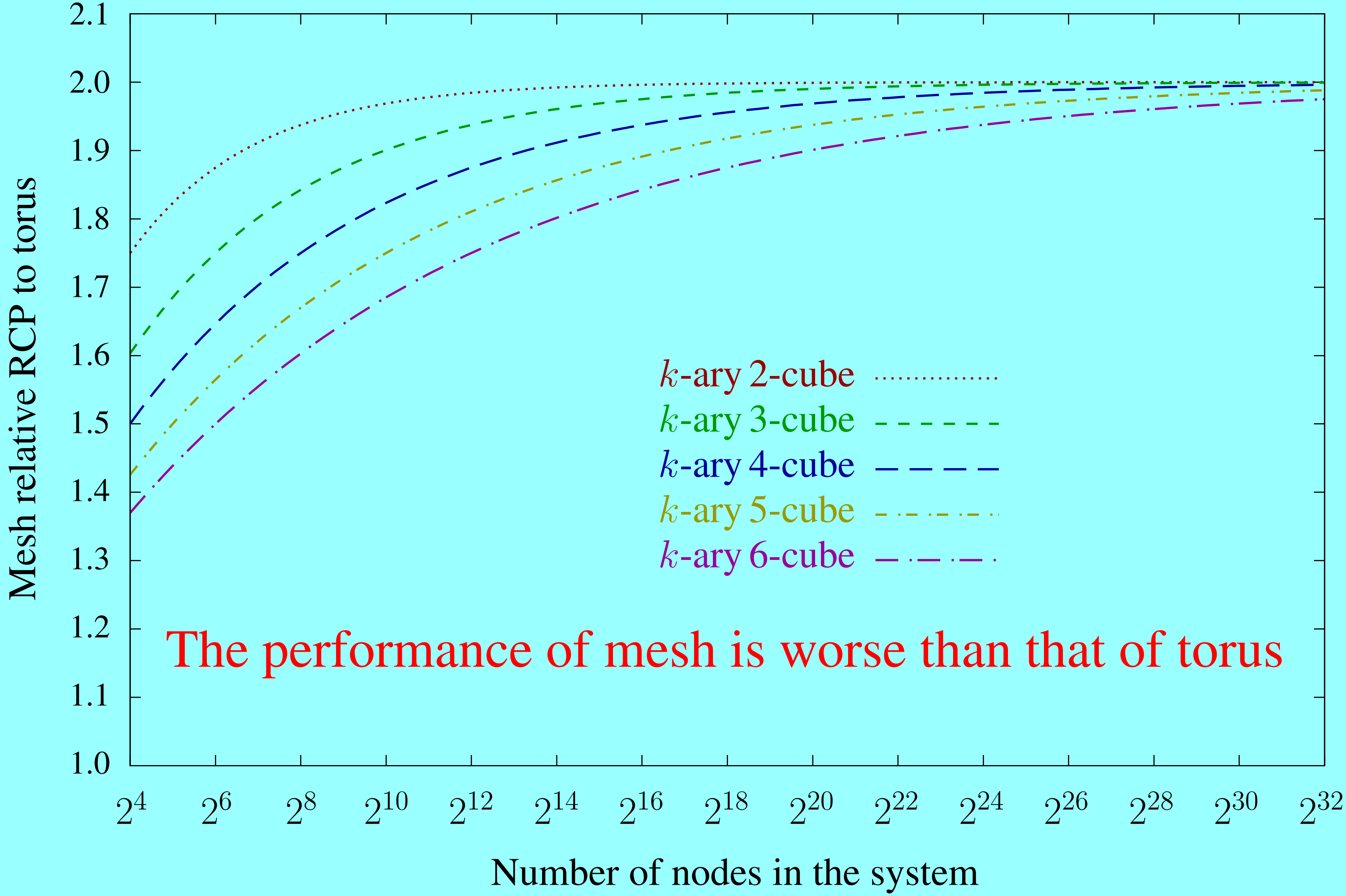
Recommended Bidirectional Tori with $p = 1$

N	n	k	d	D	RCP	λ
121	2	11	4	10	0.576	2.0
256	2	16	4	16	0.617	2.0
343	3	7	6	9	0.794	1.0
1,000	3	10	6	15	0.768	1.5
3,375	3	15	6	21	0.543	2.0
4,913	3	17	6	24	0.545	2.0
14,641	4	11	8	20	0.683	1.5
16,807	5	7	10	15	0.782	1.0
50,625	4	15	8	28	0.525	2.0
117,649	6	7	12	18	0.779	1.0
161,051	5	11	10	25	0.674	1.5
248,832	5	12	10	30	0.742	1.5
759,375	5	15	10	35	0.514	2.0
1,771,561	6	11	12	30	0.668	1.5
2,476,099	5	19	10	45	0.518	2.0
11,390,625	6	15	12	42	0.507	2.0
47,045,881	6	19	12	54	0.728	2.0

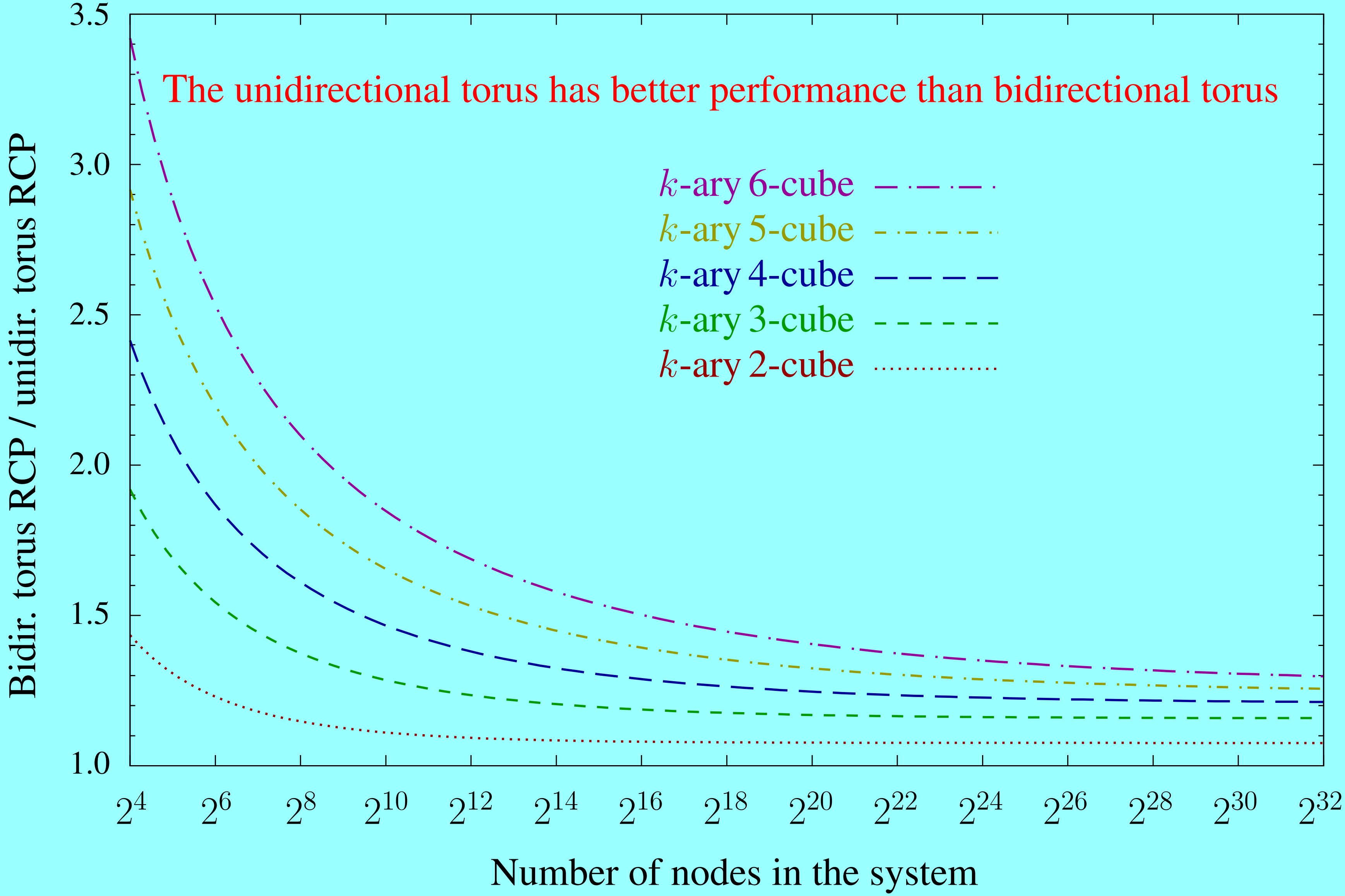
RCP of Mesh ($p = 1$, $\lambda = 1.5$)



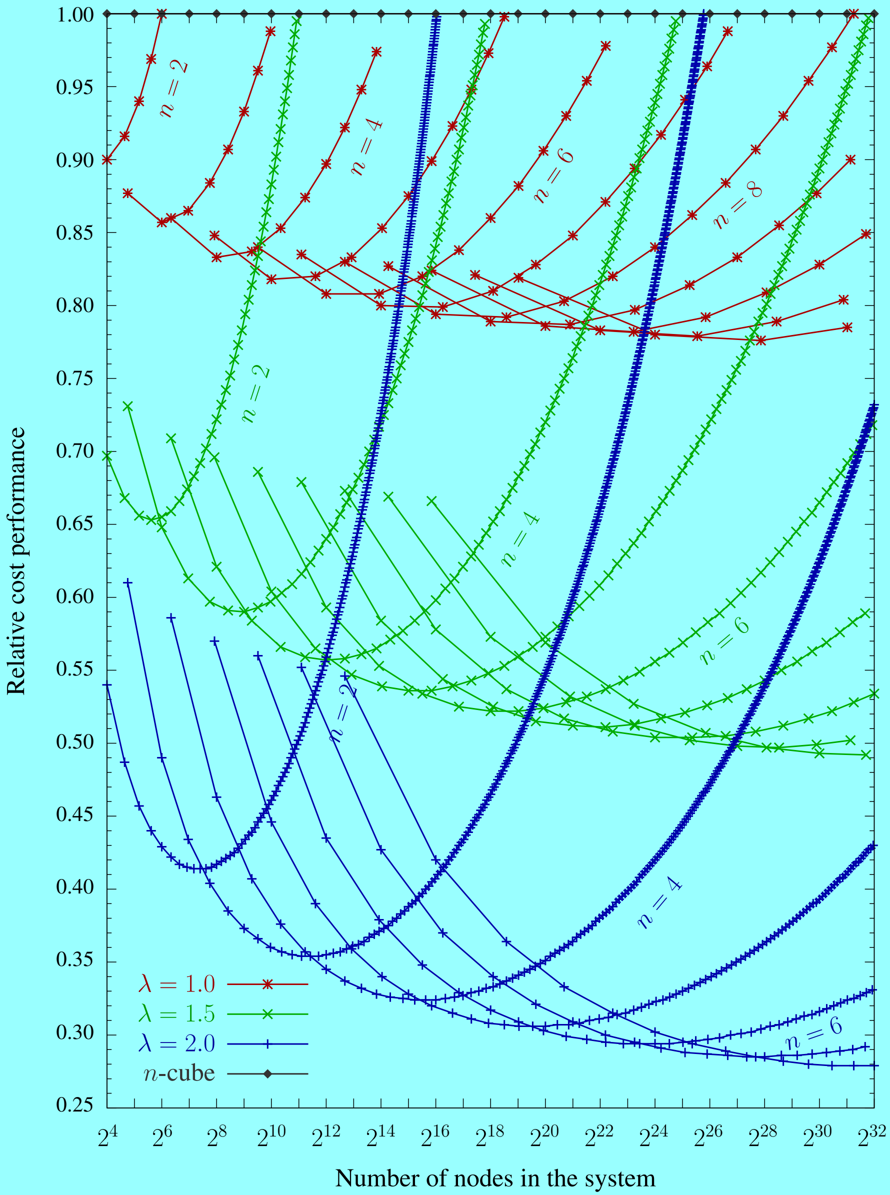
Dividing Mesh RCP by Torus RCP



Improvement of Unidirectional Torus



RCP Comparison for Unidirectional Torus



Recommended Undirectional Tori with $p = 1$

N	n	k	d	D	RCP	λ
196	2	14	2	26	0.414	2.0
256	4	4	4	12	0.833	1.0
512	3	8	3	21	0.590	1.5
1,024	5	4	5	15	0.818	1.0
2,744	3	14	3	39	0.354	2.0
3,375	3	15	3	42	0.354	2.0
4,096	4	8	4	28	0.557	1.5
15,625	6	5	6	24	0.808	1.0
32,768	5	8	5	35	0.536	1.5
50,625	4	15	4	56	0.324	2.0
59,049	5	9	5	40	0.536	1.5
262,144	6	8	6	42	0.522	1.5
531,441	6	9	6	48	0.522	1.5
759,375	5	15	5	70	0.306	2.0
1,048,576	5	16	5	75	0.306	2.0
7,529,536	6	14	6	78	0.294	2.0
24,137,569	6	17	6	96	0.294	2.0

Summary

- The k-ary n-cube has been deeply investigated and widely adopted in real supercomputer designs
- We proposed an analytical model for evaluating the relative cost performance to hypercube

$$RCP = ((d+p)^{\lambda} D) \,/\, ((\log_2 N + p)^{\lambda} \log_2 N)$$

- By using this model, we can configure the k-ary n-cube to achieve high performance at low cost.
- We also investigated k-ary n-dimensional mesh and unidirectional k-ary n-dimensional torus
- The unidirectional k-ary n-dimensional torus is better than that of the bidirectional k-ary n-dimensional torus